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Environmental Efficiency Measurement and the Materials Balance Condition Reconsidered

1. Introduction

The productivity and efficiency analysis literature has become engaged with production processes in which undesirable outputs (pollutants) are jointly produced with desirable outputs. Consequently, the appropriate way to model undesirable outputs has been debated. Among the most popular approaches are to incorporate pollutants as freely disposable inputs (Baumol & Oates, 1975; Pittman, 1981) or weakly disposable outputs (Färe et al., 1989; Färe et al., 2005) in the production model. They have recently been criticized for not complying with the materials balance condition (e.g. Coelli et al., 2007; Rødseth, 2013; Rødseth & Romstad, 2014), a law of physics that explains the generation of pollution in many conventional production processes.

Coelli et al. (2007) introduced a new environmental efficiency measure that ensures consistency with the materials balance condition. Unlike the above-mentioned modeling approaches, Coelli et al. (2007) do not treat pollutants as inputs or outputs to be included in the production model, but utilize material flow coefficients to identify the input mix that results in the minimal material inflow required to produce a certain bundle of desirable outputs. Their material inflow minimization problem is parallel to the well-known cost minimization problem in economics, and Coelli et al.'s (2007) efficiency measure therefore corresponds to Farrell's (1957) cost efficiency measure. Economic-environmental trade-offs can further be assessed by comparing the material inflow minimization to cost minimization; see e.g. van Meensel et al. (2010).

This note takes up an issue that was not satisfactory dealt with by Coelli et al. (2007), namely the role of pollution control in environmental efficiency measurement. More specifically, it shows that Coelli et al.'s efficiency measure may provide biased efficiency rankings in cases where pollution control is a common compliance strategy. The reason is that their approach only cares about reallocation (or minimization) of material inputs to minimize *uncontrolled* emissions, and neglects the fact that *additional* inputs can be used to *control* pollutants. Pollution control, e.g. end-of-pipe abatement, is quite common in many of the industries which Coelli et al.'s model has been applied to – e.g. electricity (Welch & Barnum, 2009) and agriculture (Coelli et al., 2007) – although it has not been taken into account in these empirical analyses.

Rødseth (in review) recently introduced the axioms "weak G-disposability" and "output essentiality" and showed that they allow the specification of a production model that includes pollutants and, at the same time, is consistent with the materials balance condition. Hampf and

Rødseth (2015) showed how Rødseth's theoretical production model can be implemented using Data Envelopment Analysis (Charnes et al., 1978), abbreviated DEA. This approach makes up the foundation for the current note, which introduces a new environmental efficiency measure that rewards emission reductions by pollution control. Coelli et al.'s (2007) environmental efficiency measure is a special case of the proposed efficiency measure.

2. Background

The materials balance condition implies that the amount of materials bound in inputs must equal the amount of materials bound in the intended outputs and residuals (in our case, the pollutant). Let $x \in \Re_+^N$ denote a vector of inputs, $y \in \Re_+^M$ denote a vector of desirable outputs, and let the pollutant be denoted $b \in \Re_+$ [1]. Further, let $u \in \Re_+^N$ be a vector of material flow coefficients for inputs and let $v \in \Re_+^M$ be a vector of material flow coefficients for intended outputs. The "production function" for the pollutant (i.e., the materials balance condition) can then be represented by:

$$b = \sum_{n=1}^{N} u_n x_n - \sum_{m=1}^{M} v_m y_m \tag{1}$$

Eq. 1 is a representation of *uncontrolled* emissions.

Coelli et al. (2007) note that if the desirable outputs are considered fixed, it follows from Eq. 1 that the pollutant is minimized when the material inflow is minimized. Formally, they solve the minimization problem $\min_x \left\{ \sum_{n=1}^{N} u_n x_n : (x, y) \in T \right\}$, where *T* is the traditional neo-classical technology. I consider this minimization problem highly relevant for modeling well-known compliance strategies such as input substitution (e.g. switching from high-sulfur to low-sulfur fuels) and efficiency improvements. Unfortunately, as argued by Rødseth (2013), these compliance strategies do not cover all of the decision making units' (DMUs') available strategies. For some DMUs it may be less costly to use *additional* resources to clean up their uncontrolled emissions than to substitute cheap high-polluting inputs with costly low-polluting inputs. In this case, the minimization of *controlled* emissions:

[1] I consider only one bad output to ensure consistency with Coelli et al.'s (2007) approach. However, the model outlined in section 3.1 can be generalized to multiple pollutants.

$$b = \sum_{n=1}^{N} u_n x_n - \sum_{m=1}^{M} v_m y_m - a \quad (2)$$

is the relevant objective, not the minimization of uncontrolled emissions. Here, $a \in \Re_+$ denotes the amount of uncontrolled emissions that is reduced by pollution control[2].

The problem with ignoring pollution control efforts in environmental efficiency measurement is illustrated by the following hypothetical example on manure transport in pig finishing (i.e., the process of raising a piglet to a slaughter hog). The main inputs to pig finishing are piglets and feed, while capital and labor are minor inputs (Coelli et al., 2007). Piglets and feed provide nitrogen inflows to the pig finishing process, but only about 36 percent of the inflows are retained in the saleable meat (Lauwers, 2009). The residual nitrogen is an unintended product, generally contained in manure.

The manure byproduct can be collected and transported to other farms for use as a fertilizer. The nitrogen surplus remaining on the farm after the manure transport is thus its controlled nitrogen byproduct. Manure transport is a costly operation that requires both capital and labor inputs. To accommodate this feature, I construct a hypothetical numerical example in which the farms' involvement in manure transport are generally reflected by their capital and labor intensities. I report the artificial feed, piglets, and meat data in 100 kilos, and use the material flow coefficients from Coelli et al. (2007) to calculate the farms' nitrogen surpluses. Table 1 provides an overview of the sample.

Table 1: *Hypothetical sample*

Farm	Feed	Piglet	Labor	Capital	Meat	Controlled emissions	Uncontrolled emissions
A	21.0	2.0	50.0	100.0	10.0	16.0	16.7
B	21.0	2.0	60.0	130.0	10.0	11.0	16.7
C	24.0	2.0	60.0	130.0	10.0	20.0	20.4
D	24.0	2.0	90.0	160.0	10.0	10.0	20.4
E	24.0	1.5	50.0	100.0	10.0	19.0	19.8

[2] Note that Eq. 2 does not distinguish between the resources used to control pollution and to produce intended outputs, in contrast to e.g. Pethig (2006). My paper is in the spirit of Färe et al. (2007) who address the implications of pollution abatement for productivity measurement using a production model comprising inputs and good and bad outputs, with no explicit reference to the pollution control technology. That is, since pollution control efforts are manifested as reductions in emissions, Färe et al. (2007) examine abatement efficiencies by considering the relationship between the input consumption and controlled emissions. In cases where the amount abated and the inputs used for intended production and pollution control can readily be identified, one might consider extending the traditional production model with an abatement output (Coelli et al., 2007) or to model the polluting production process using network technologies (Färe et al, 2013; Hampf, 2014). This information can only be obtained for a very limited number of case studies and/or may result in small samples, suggesting that the approach proposed in this paper is more useful for empirical research.

Note that the nitrogen flow coefficients for capital and labor are zero, meaning that they are treated as free inputs in Coelli et al.'s (2007) nitrogen minimization problem. Since all farms produce the same amount of meat, its solution can readily be inferred from table 1 by identifying the farm(s) consuming the mix of feed and piglets that results in the minimal nitrogen inflow and thus uncontrolled emissions. Ranked according to this criterion, farms A and B are environmental efficient; cf. the column "Uncontrolled emissions" in Table 1. The question is, however, whether this is a *fair* ranking of environmental performance. Farms B and D exhibit the lowest controlled nitrogen surpluses in the sample, yet farm D is considered environmentally inefficient by Coelli et al.'s measure. Farms A and B receive the same efficiency score (i.e., 1), although B's controlled emissions are lower than A's controlled emissions. Farm B is thus not rewarded for its efforts to control nitrogen emissions. The reason for this result is Coelli et al.'s choice of objective function, namely the minimization of nitrogen inflows to the production process. The additional capital and labor that farm B spend on manure transport relative to farm A are, in principle, seen as unproductive, since they do not contribute to improving the farm's environmental efficiency score. This suggests that a highly relevant environmental-economic trade-off (cf. Van Meensel et al. (2010)) is neglected for the hypothetical sample, as well as for real-life samples where pollution control is a common compliance strategy.

3. Rewarding pollution control in environmental efficiency measurement

3.1. Methods

The purpose of this section is to introduce a new efficiency measure that rewards pollution control efforts. My approach builds on Rødseth (in review) and Hampf and Rødseth (2015), who recently introduced a new production model that incorporates pollutants but at the same time is consistent with the materials balance condition. This model replaces 1) the traditional axioms of free disposability of inputs and outputs by weak G-disposability and 2) the inactivity axiom by output essentiality for the pollutant[3]. The weak G-disposability axiom ensures that the disposal of inputs and outputs takes place according to the materials balance principle, while the output essentiality axiom satisfies the second law of thermodynamics by enforcing that the consumption of material inputs always is accompanied by byproducts.

Hampf and Rødseth (2015) showed how the theoretical production model can be implemented empirically by DEA. To ensure output essentiality, each DMU in the dataset must use a strictly

[3] Besides the disposability and inactivity axioms, the other axioms of the neo-classical production model are found consistent with the materials balance condition. See Färe and Primont (1995) for an overview of these axioms. The applicability of the convexity axiom is particularly important for this paper, since it makes up the foundations of DEA together with the disposability axioms.

positive amount of the polluting inputs and produce a strictly positive amount of the bad output. Weak G-disposability is implemented by restricting the slack variables ($\varepsilon_x \in \Re_+^N, \varepsilon_y \in \Re_+^M, \varepsilon_b \in \Re_+$) in the DEA programming problem according to the materials balance condition, i.e. by imposing the constraint $\sum_{n=1}^{N} u_n \varepsilon_{xn} + \sum_{m=1}^{M} v_m \varepsilon_{ym} = \varepsilon_b$. In words, the changes in emissions due to changes in inputs ($\sum_{n=1}^{N} u_n \varepsilon_{xn}$) and desirable outputs ($\sum_{m=1}^{M} v_m \varepsilon_{ym}$) must equal the change in pollution (ε_b) when inputs and outputs are disposed (for a given level of pollution control).

Assume there are *l=(1,..,L)* DMUs in the dataset. The input vector is partitioned into *S* material (i.e., polluting; denoted *P*) and *N-S* nonmaterial (i.e., nonpolluting; denoted *NP*) inputs, associated with material flow coefficients that are positive and zero, respectively. Each DMU uses inputs $x^l = \left(x_P^l, x_{NP}^l\right) = \left(x_1^l, .., x_S^l, x_{S+1}^l, .., x_N^l\right) \in \Re_+^N$ to produce the desirable outputs $y^l = \left(y_1^l, .., y_M^l\right) \in \Re_+^M$ and an undesirable output $b^l \in \Re_+$. Let λ^l, *l=(1,..,L),* define the intensity variables. On the basis of the constant returns to scale (CRS) DEA model, consider three minimization problems that define the minimal amount of pollution for given desirable outputs and for 1) fixed inputs, 2) fixed nonmaterial inputs, and 3) variable inputs, respectively, for DMU *l'*.

Minimization problem 1: *fixed inputs*	**Minimization problem 2:** *fixed nonmaterial inputs*	**Minimization problem 3:** *variable inputs*
$b_F^* = \min_{\varepsilon,b,\lambda} \{ b \ : \ \sum_{l=1}^{L} \lambda^l y_m^l - \varepsilon_{ym} = y_m^{l'},\ m=1,..,M$ $\sum_{l=1}^{L} \lambda^l b^l + \varepsilon_b = b$ $\sum_{l=1}^{L} \lambda^l x_n^l + \varepsilon_{xn} = x_n^{l'},\ \text{n}=1,..,S$ $\sum_{l=1}^{L} \lambda^l x_n^l + \varepsilon_{xn} = x_n^{l'},\ \text{n}=S+1,.,N$ $\sum_{n=1}^{N} u_n \varepsilon_{xn} + \sum_{m=1}^{M} v_m \varepsilon_{ym} - \varepsilon_b = 0$ $\lambda, \varepsilon_y, \varepsilon_b, \varepsilon_x \geq 0 \}$	$b_{QF}^* = \min_{\varepsilon,b,\lambda,x_P} \{ b \ : \ \sum_{l=1}^{L} \lambda^l y_m^l - \varepsilon_{ym} = y_m^{l'},\ m=1,..,M$ $\sum_{l=1}^{L} \lambda^l b^l + \varepsilon_b = b$ $\sum_{l=1}^{L} \lambda^l x_n^l + \varepsilon_{xn} = x_n,\ \text{n}=1,..,S$ $\sum_{l=1}^{L} \lambda^l x_n^l + \varepsilon_{xn} = x_n^{l'},\ \text{n}=S+1,.,N$ $\sum_{n=1}^{N} u_n \varepsilon_{xn} + \sum_{m=1}^{M} v_m \varepsilon_{ym} - \varepsilon_b = 0$ $\lambda, \varepsilon_y, \varepsilon_b, \varepsilon_x \geq 0 \}$	$b_V^* = \min_{\varepsilon,b,\lambda,x} \{ b \ : \ \sum_{l=1}^{L} \lambda^l y_m^l - \varepsilon_{ym} = y_m^{l'},\ m=1,..,M$ $\sum_{l=1}^{L} \lambda^l b^l + \varepsilon_b = b$ $\sum_{l=1}^{L} \lambda^l x_n^l + \varepsilon_{xn} = x_n,\ \text{n}=1,..,S$ $\sum_{l=1}^{L} \lambda^l x_n^l + \varepsilon_{xn} = x_n,\ \text{n}=S+1,.,N$ $\sum_{n=1}^{N} u_n \varepsilon_{xn} + \sum_{m=1}^{M} v_m \varepsilon_{ym} - \varepsilon_b = 0$ $\lambda, \varepsilon_y, \varepsilon_b, \varepsilon_x \geq 0 \}$

where the subscripts *F, QF,* and *V* denote fixed, quasi-fixed, and variable (inputs), respectively, referring to the different degrees of freedom to determine the input mix. As will subsequently be explained, the programming problems enable assessing the contributions of various abatement strategies to minimizing pollution, given that DMU l' upholds its current production of desirable outputs. Their solutions make up the foundations of the following efficiency measure and its decompositions:

$$\frac{b_V^*}{b^{l'}} = \frac{b_F^*}{b^{l'}} \frac{b_{QF}^*}{b_F^*} \frac{b_V^*}{b_{QF}^*} \tag{3}$$

The overall efficiency measure and its components take values smaller or equal to 1, where 1 indicates that the DMU under consideration is efficient. The overall measure evaluates the DMU's possibility to reduce its controlled emissions given its production of desirable outputs, while the components assess the contributions of different abatement strategies to the overall emission reduction. More specifically, the overall efficiency measure decomposes into 1) emission reductions for given inputs $\left(b_F^* / b^{l'}\right)$, 2) additional emission reductions by minimizing the material inflow $\left(b_{QF}^* / b_F^*\right)$, and 3) further emission reductions by reallocating material and nonmaterial inputs $\left(b_V^* / b_{QF}^*\right)$. The first component is a measure of emission reduction due to technical efficiency improvements. To see this, consider the short-run Farrell (1957) efficiency measure which minimizes pollution for given inputs and desirable outputs, i.e., $TE = \inf\left\{\theta : \left(x^{l'}, y^{l'}, \theta b^{l'}\right) \in T\right\}$. The solution to this minimization problem is $\theta b^{l'} = b_F^* \Leftrightarrow \theta = b_F^* / b^{l'}$, corresponding to my efficiency measure. Hence, I dub the component $b_F^* / b^{l'}$ "technical environmental efficiency"[4].

The remaining components are dubbed "material and nonmaterial allocative environmental efficiencies", referring to that material and nonmaterial inputs must be reallocated to further improve environmental efficiencies. The material allocative efficiency component resembles the allocative efficiency component of Coelli et al.'s (2007) efficiency measure[5], and concerns the minimization of material inflows as a strategy to reduce emissions. The nonmaterial allocative

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[4] Eq. 1 (uncontrolled emissions) does in general not allow for inefficiency in pollution generation since the sum of the materials bound in desirable and undesirable outputs must amount to the aggregate material content of the inputs. Intuitively, the technical pollution efficiency component therefore concerns differences in pollution control performances (cf. Eq. 2).

[5] An important distinction is of course that Coelli et al. (2007) do not consider nonmaterial inputs to be fixed when minimizing uncontrolled emissions.

efficiency component captures the contributions of nonmaterial inputs to reduce emissions, e.g. by replacing material inputs or playing pivotal roles in pollution control.

An important difference between the new environmental efficiency measure and Coelli et al.'s (2007) efficiency measure is that the former evaluates a DMU's potential to reduce its controlled emissions while the latter focuses on its potential to reduce uncontrolled emissions. Both measures decompose the contributions of technical efficiency improvements and material inflow allocative efficiencies to pollution minimization[6]. However, Coelli et al. (2007) do not emphasize the contributions of nonmaterial inputs to reduce emissions. Their method will identify DMUs that consume little materials as efficient, despite their reliance on nonmaterial inputs (Dakpo et al., 2015).

3.2. Results

Table 2 reports the environmental efficiency scores obtained by implementing the DEA models from Sect 3.1 on the hypothetical sample from Table 1.

Table 2: *Efficiency scores*

Farm	$b^{l'}$	b_F^*	b_{QF}^*	b_V^*	Technical efficiency	Material allocative efficiency	Nonmaterial allocative efficiency	Environmental efficiency	Coelli et al's measure
A	16.00	16.00	16.00	10.00	1.00	1.00	0.63	0.63	1.00
B	11.00	11.00	11.00	10.00	1.00	1.00	0.91	0.91	1.00
C	20.00	14.72	11.00	10.00	0.74	0.75	0.91	0.50	0.88
D	10.00	10.00	10.00	10.00	1.00	1.00	1.00	1.00	0.88
E	19.00	19.00	16.00	10.00	1.00	0.84	0.63	0.53	0.90

The new environmental efficiency measure considers farm D to be the only environmental efficient unit in the sample, despite that its nitrogen inflow is higher than those of farms A and B. This results because the new efficiency measures compares the farms in terms of their controlled emissions, rewarding efforts to clean up emissions. In contrast, as previously noted, farms A and B are considered environmental efficient by Coelli et al.'s measure, because their material inflows (i.e., their uncontrolled emissions) are the smallest in the sample.

The new efficiency measure considers farms A, B, D, and E technical environmental efficient. Farms A and E's consumption of capital and labor in the lowest sample, and their abatement efforts are virtually zero. There are no other units in the sample that consume similar amounts of nonmaterial inputs and simultaneously control their emissions, leading to the result that A and E

[6] Note that the two components are not identically defined across the models. E.g., Coelli et al.'s (2007) technical efficiency measure concerns material input reductions, while I measure technical efficiency keeping inputs fixed.

are technical efficient. Farms B and D are relatively more capital and labor intensive than A and E, and both farms undertake manure transport. Farm C, on the other hand, is comparable to farm B in terms of its usage of nonmaterial inputs, yet it does not reduce its emissions by manure transport. The DEA model thereby considers farm C's nonmaterial inputs wasted, in the sense that it could obtain similar emission reductions as farm B (i.e., 16.7 – 11.0 = 20.4 – 14.7 = 5.7) by becoming technical efficient.

Farms C and E are found material allocative environmental inefficient. Farm C consumes similar amount of nonmaterial inputs and produces the same amount of pig meat as farm B, yet its feed intake is higher than farm B's intake. Farms A and E are also comparable in terms of nonmaterial inputs and meat, but farm E's nitrogen inflow is higher than A's inflow.

The nonmaterial allocative environmental efficiency component compares all farms to farm D. Note that farms A and B, which are perceived environmental efficient by Coelli et al.'s (2007) measure, receive nonmaterial allocative efficiency scores of 0.63 and 0.91, respectively. The former is not involved in manure transport, while the latter does not exhibit as extensive pollution control as farm D. The numerical example thus clearly illustrates that Coelli et al.'s efficiency measure ignores the allocation of resources to pollution control. It is therefore likely to underestimate environmental efficiencies in industries where pollution control is a common compliance strategy.

A remaining question is how the new efficiency measure would compare to Coelli et al.'s efficiency measure if none of the DMUs in the sample control pollution. Intuitively, the two measures should coincide; however, this may not always be the case. The reason is that in some cases (such as pig finishing) the desirable outputs are also important for the materials balance accounting. This occurs when the byproducts (the pollutants) are recuperated or retained in the desirable outputs.

Proposition: *If 1) none of the DMUs control pollutants and 2) the material flow coefficients for the desirable outputs are zero, then the new efficiency measure and Coelli et al.'s (2007) efficiency measure coincide.*

The formal proof is in the Appendix (supplementary material).

4. Summary and conclusions

This note has introduced a new environmental efficiency measure that is consistent with the materials balance condition. The new efficiency measure is compared to Coelli et al.'s (2007) well-known environmental efficiency measure to show that the latter is likely to underestimate environmental efficiencies in industries where pollution control is widespread. The reason is that Coelli et al.'s (2007) approach only considers input minimization and substitution as possible compliance strategies, and ignores that additional resources may be used to clean up pollutants. The new approach to environmental efficiency measurement rewards pollution control as well as input minimization and substitution. As opposed to Coelli et al.'s approach, the new measure emphasizes minimization of *controlled* emissions for given desirable outputs rather than minimization of *uncontrolled* emissions.

With the aid of a simple numerical example I have illustrated that my approach is well suited for analyzing environmental-economic trade-offs where additional resources are employed to clean up pollutants. I build on a production model that ensures a strict technical relationship between the consumption of material inputs and the pollution generation according to the materials balance condition. Nonmaterial inputs are not influenced by this relationship, but are substitutes for material inputs in the conventional sense. This feature allows me to model the costs of pollution control activities: by substituting material inputs for (costly) nonmaterial inputs, the amount of pollution reduces correspondingly.

While the stylized numerical example effectively illustrates the differences between Coelli et al.'s approach and the new approach, it says little about their actual implications for applied research. Future research should address this issue by comparing available environmental efficiency methods using real data.

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