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Axioms of a Polluting Technology: A Materials Balance Approach

Abstract: This paper aims to present an economic model characterized by a set of axioms that are consistent with the laws of thermodynamics. Two new axioms – *weak G-disposability* (i.e. weak directional disposability) and *output essentiality* – are introduced to satisfy the materials balance principle and the entropy law, respectively. The axiomatic production model is compared to other well-known production models that account for the joint production of good and bad outputs to illustrate the advantages of the new modeling approach.

Keywords: Axiomatic production model; Materials balance principle; *G*-disposability; Essentiality; Environmental Efficiency

JEL-codes: D24; Q57

1. Introduction

Economic models are frequently criticized for neglecting physical laws and the role that natural resources play in production¹. Ayres and Kneese (1969) were the first to examine the economic implications of the laws of thermodynamics. They argued that environmental degradation are associated with production and consumption residuals that arise due to the materials balance principle (the first law of thermodynamics). Yet, in economics, the interest in correctly describing material flows remained moderate (Pethig, 2003). Relevant branches such as production and environmental economics have ignored the materials balance principle for a long time (Lauwers, 2009). Only recently, there has been a renewed interest in the laws of thermodynamics in economics.

In the area of environmental and ecological economics, some recent studies – including Ruth (1995), Murty and Russell (2002), Krysiak and Krysiak (2003), Baumgärtner (2004), Pethig (2006), Ebert and Welsch (2007), Førsund (2009), and Roma and Pirino (2009) – account for energy flows, material flows, and end-of-pipe abatement. With a few exceptions where the (economic) production functions are derived from the physical models, the studies integrate the neo-classical technology (or the economic production function) into a larger technology that includes energy and material balances. The results are complex models that generally require substantial amounts of information for empirical analysis. In other words, increased physical realism comes at the expense of model tractability and manageable data requirements.

Frisch (1965) uses the example of silver spoon production to illustrate that any type of analysis cannot undertake a complete characterization of all physical processes underlying even a (presumably) simple production process. Consequentially, he concludes that an appropriate analytic approach must select certain factors whose effects are studied in more detail.

Recently, the materials balance principle has also arisen as a topic in the literature on frontier-based eco-efficiency models (Färe et al., 2013), i.e., economic models that offer the possibility to examine the joint production of good and bad outputs. These models build on microeconomic production theory, and are thus primarily concerned with inputs that are in some sense economically scarce and over which the entrepreneur exercises effective control (Chambers, 1988). As a result, frontier-based models are highly tractable with manageable data requirements compared to the before-mentioned models from environmental and ecological economics. Hence, they are more appropriate for applied economic analysis. This paper aims to establish a frontier-based production model that is in line with the physical production

constraints dealt with in the environmental and ecological economics literature, but at the same time maintains the desirable simplicity of the production analysis approach.

Lauwers (2009) identifies three classes of frontier-based eco-efficiency models; the frontier eco-efficiency (FEE) models, the environmentally adjusted production efficiency (EAPE) models, and the materials balance principle (MBP) adjusted production efficiency method. The FEE models do not account for conventional inputs and outputs, but compare ecological and economic outcomes. These models are not treated further in this paper. I am primarily concerned with models that describe the conversion of inputs into good and bad outputs, since the materials balance principle is readily applicable in this setting.

The EAPE class comprises economic production models extended by pollutants², while the MBP method is based on the traditional economic production model that omits pollutants, but uses material flow coefficients (i.e., the materials balance principle) to provide an explicit link between the production technology and the environmental outcome. These two classes of models have pros and cons. The EAPE models are criticized for being inconsistent with the materials balance principle (Coelli et al., 2007; Hoang and Coelli, 2011) and for being unsuitable for case studies where end-of-pipe abatement is not among the preferred abatement strategies of the decision making units under consideration (Rødseth and Romstad, 2014). The MBP method's use of the conventional economic production model may, on the other hand, lead to biased efficiency measurement when decision making units spend resources on cleaning up pollution (Färe et al., 2007).

The current paper uses the EAPE models as a stepping stone, but considers how their underlying model properties (i.e. axioms) can be modified to secure consistency between the economic production model and the laws of thermodynamics. By consistency, the proposed EAPE model not only inherits the desirable features of the MBP method, but also extends and

enriches this class of frontier-based eco-efficiency models. Using simple numerical examples, I illustrate that my approach is appropriate for modeling recuperation and cleanup of pollution.

The paper starts by examining the central axioms of the most popular EAPE model – weak disposability of good and bad outputs, null-jointness, inactivity, and free disposability of inputs – and shows that they, contrary to the beliefs in the literature (Coelli et al., 2007; Hoang and Coelli, 2011), are consistent with the materials balance principle *if* end-of-pipe abatement can be adapted to maintain material balance. However, this is not a satisfactory result since the requirements on end-of-pipe abatement efforts are strong and, generally, physically unattainable. The current paper therefore introduces an alternative axiomatic production model that does not rely on changes in end-of-pipe abatement in order to comply with the laws of thermodynamics. First, the paper introduces a weaker form of G-disposability (Chung, 1996), weak G-disposability, and shows that the weak G-disposability axiom secures consistency between the axiomatic production model and the materials balance principle. Second, a new type of essentiality, output essentiality, is introduced in the form of an axiom. Output essentiality rules out cases in which bad outputs are zero for positive entries of pollution-generating inputs. It is clearly in line with the second law of thermodynamics, but it rules out the inactivity and null-jointness axioms. Other standard axioms of the neo-classical production model – non-emptiness, no free lunch, closedness, boundedness, and convexity – are found to apply in the material balance setting.

This paper is structured as follows: The next section discusses the laws of thermodynamics and presents two numerical examples. Section 3 introduces the axiomatic production model framework and discusses the axioms of Färe et al.'s (1989; 2005) EAPE model, while section 4 establishes a set of axioms that are in line with the laws of thermodynamics. Section 5

compares my production model to the established EAPE and MBP approaches, while section 6 concludes.

2. The laws of thermodynamics

The first law of thermodynamics - or the materials balance principle³ - is a well-known law of physics that in simple terms says “what goes into the production process must also come out” (Coelli et al., 2007). For large economic systems, the materials balance principle implies that all matter entering the economic system ultimately ends up in the environment. However, materials balance is an additive condition, and the principle can be transposed to narrowly defined sub-systems of the overall system (Lauwers, 2009), e.g., nutrient balances in agriculture. Let $x \in \mathfrak{R}_+^N$ denote a vector of inputs, $y \in \mathfrak{R}_+^M$ denote a vector of good outputs, and let the residual output be considered a bad output⁴ denoted $b \in \mathfrak{R}_+$. Further, let $u \in \mathfrak{R}_+^N$ be a vector of material flow coefficients for inputs⁵, and let $v \in \mathfrak{R}_+^M$ be a vector of material flow coefficients for outputs. The materials balance principle is then defined by equation 1:

$$b = ux - vy \tag{1}$$

Equation 1 represents the producer’s uncontrolled emissions. That is, it captures the amount of the bad output that results when a certain amount of inputs, x , is used to produce a certain amount of good outputs, y . Clearly, an increase in the efficiency of converting inputs into good outputs, or a downscaling in the use of inputs associated with large material flow coefficients, for example by substituting high-polluting inputs with low-polluting inputs, reduce uncontrolled emissions. The bad output may also be reduced by means of end-of-pipe abatement, which involves diminishing the bad output by converting it into different

byproducts instead of reducing the uncontrolled emissions themselves. The use of calcium oxide (that react with sulfur dioxide to form calcium sulfite) to diminish sulfur dioxide emissions from electric power plants is one example of end-of-pipe abatement. Mathematically, end-of-pipe abatement is represented by subtracting $a \in \mathfrak{R}_+$ from equation 1⁶. The release of the bad output after end-of-pipe abatement is called controlled emissions:

$$b = ux - vy - a \tag{2}$$

Notice from expressions 1 and 2 that the materials balance principle does not preclude the possibility that the bad output is zero, even when positive amounts of inputs are used to produce positive amounts of desirable outputs. However, the second law of thermodynamics, often called the entropy law, implies that materials can only be incompletely transformed into good outputs. Consequentially, it rules out that the bad output can be zero for positive entries of pollution-generating (material) inputs (Baumgärtner et al., 2001; Ebert and Welsch, 2007).

2.1.Numerical examples

The purpose of this section is to provide two numerical examples – one with and one without end-of-pipe abatement – that allow me to illustrate the properties of the materials balance principle and its implications for economic modeling. I will return to these examples throughout the paper. They will serve as a benchmark to which both my new production model and the established EAPE and MBP models will be compared.

The nitrogen metabolism in pig finishing, adopted from Coelli et al. (2007) and Lauwers (2009), is my first example. Pig finishing is the process of raising a piglet to a slaughter hog, which without substantial bias from reality can be seen as a production process with two inputs

(piglets and feed) and one good output (meat) (Lauwers, 2009) that is produced jointly with an uncontrolled byproduct (mainly manure). Although there are available end-of-pipe abatement processes for manure (e.g., transport of manure to other farms that can use it as an input), I consider the uncontrolled byproduct to be a bad output. More specifically, the pig producers are assumed not to clean up the residual waste. This assumption is in line with Coelli et al. (2007) and Lauwers (2009).

Table 1

Table 2

Consider three decision making units (pig farms) that use the same amount of inputs, but differ in terms of the good outputs produced. This variation could be due to differences in feed waste or digestibility. For simplicity, I report the artificial data in 100 kilos, and use the material flow coefficients from Coelli et al. (2007) to calculate the nitrogen surplus. These coefficients are 1.17 for piglets and pig meat and 1.24 for feed. Table 1 presents the input and output data in 100 kilos, while table 2 presents the nitrogen contents in inputs and outputs, as well as the residual. Note that the nitrogen contents of the inputs (feed and piglets) sum to the nitrogen contents of the outputs (meat and residual), and further that the nitrogen from feed on average amounts to 92.5 percent of the nitrogen inflow, while the residual waste on average amounts to 63 percent of the nitrogen outflow.

The second example concerns sulfur dioxide emissions from coal-fired electricity generation, a beloved case study in the EAPE modeling literature; see e.g. Färe et al. (2005; 2013). While sulfur dioxide emissions are contingent on the sulfur content of the coal used to generate heat, the sulfur dioxide case differs from the nitrogen emission case in the sense that the saleable output (electricity) does not contain sulfur. By the materials balance principle, the sulfur inflow to the combustion process thereby inevitable ends up as a residual. About 95 percent of the fuel sulfur is converted to sulfur dioxide, while the remaining fuel sulfur is converted to sulfur trioxide and sulfate particulate.

Sulfur dioxide is among the most important contributors to acid rain, and sulfur dioxide emissions from points sources are therefore often subject to environmental regulations; see e.g. Rødseth and Romstad (2014) for a description of sulfur dioxide regulations for power plants in US. End-of-pipe abatement has proved to be a cost efficient compliance strategy in the case of stringent sulfur dioxide regulations. Abatement equipment, often called scrubbers, remove sulfur emissions from flue gases by turning them into liquid solutions, solid pastes, or powder. However, end-of-pipe abatement generally requires the employment of additional resources that contribute little or nothing to the production of electricity; see Shadbegian and Gray (2005) for an empirical study on this issue.

Table 3

I construct a simple numerical example comprising 5 electricity generating units that use two inputs (coal and capital) to produce two outputs (electricity and sulfur dioxide). A similar description of bituminous generation can be found in Mekaroonreung and Johnson (2012). Table 3 reports the artificial data, where the data on coal inputs are in 100 tons, capacity is a monetary measure, electricity is in 100 Mwhs, and sulfur dioxide emissions are in 100 tons emitted. Note that the uncontrolled emissions are equal for all units consuming the same amount of coal (since their sulfur inflows are equal), but that their controlled emissions may not be equal due to differences in end-of-pipe abatement efforts (or removal efficiencies). I construct the numerical example such that the generating units' capital shares increase with their end-of-pipe abatement efforts, thus allowing end-of-pipe abatement to be costly.

I assume that only the sulfur dioxide emissions associated with the sulfur inflow constitute a bad output from electricity generation, and thus estimate sulfur dioxide emissions using an emission factor for sulfur dioxide rather than the material flow coefficient (which also comprises the sulfur inflow to the formation of sulfur trioxide and sulfate particulate). I use equation 1 to calculate the uncontrolled emissions, assuming an emission factor of 0.04 tons of

CO₂ per 100 tons of bituminous coal, in line with the Environmental Protection Agency's recommended emission factor for bituminous coal adjusted for the average sulfur content of bituminous coal in 2012⁷.

3. Theoretical background on production analysis

Coelli et al. (2007) utilize equation 1 (uncontrolled emissions) to show that Färe et al.'s (1989; 2005) axiomatic production model – the most popular EAPE model – is inconsistent with the materials balance principle. They use the hyperbolic distance function (Färe et al., 1985) to derive this result. In a follow-up article, Hoang and Coelli (2011) restate the argument in terms of the directional distance function (Chambers et al., 1998), which is more commonly used in the EAPE literature. For the sake of clarity, it is convenient to first introduce the definition of the technology and distance functions before proceeding with the proof of Hoang and Coelli (2011).

Note that I will primarily think of the pollutant as an output in the following. When the bad is treated as an input, the axiom of free disposability (of inputs) suggests that any vector of inputs and desirable outputs may yield an unbounded amount of the bad output. This feature cannot be consistent with physical laws (Färe and Grosskopf, 2003).

The technology is the set of all technical possibilities that a producer faces. The representation of the technology that will primarily be dealt with in this paper is as follows:

$$T = \{(x, y, b) : x \text{ can produce } (y, b)\} \quad (3)$$

Two other representations of the technology will also be considered. The first is the output set $P(x)$, which defines technical output possibilities for each input vector.

$$P(x) = \{(y, b) : (x, y, b) \in T\} \quad (4)$$

Furthermore, the technology can be represented by the input set $L(y, b)$, which defines required inputs for each output vector.

$$L(y, b) = \{x : (x, y, b) \in T\} \quad (5)$$

In empirical studies, the technology is estimated from data using either econometrics or programming models. To extract the technology from data, a set of axioms – assumptions about the production possibilities that a producer faces – is required. These axioms, in turn, determine the properties of the technology and govern the model’s estimates of production possibilities. Clearly, these estimates should be in line with the physical limits that the producer faces.

The traditional “neo-classical” axiomatic production model embodies the following characteristics:

- (i) T is nonempty
- (ii) T is closed
- (iii) For every finite x , T is bounded from above (the output sets are bounded)
- (iv) Inactivity is feasible, i.e. $(x, 0_M, 0) \in T$ for $x \in \mathfrak{R}_+^N$
- (v) There is no free lunch, i.e. if $(y, b) \geq 0_{M+1}$, then $(0_N, y, b) \notin T$
- (vi) T is a convex set
- (vii) Inputs are freely disposable,
i.e. if $(x, y, b) \in T$ and $x' \geq x$, then $(x', y, b) \in T$
- (viii) Outputs are freely disposable,
i.e. if $(x, y, b) \in T$ and $(y', b') \leq (y, b)$, then $(x, y', b') \in T$

The first axiom secures the existence of at least one feasible input-output combination. Axiom (ii) secures that there are “no holes” in the boundary of the technology set, while axiom (iii) implies that only finite amounts of outputs can be produced by each finite input vector.

Together, axioms (ii) and (iii) secure the existence of a “maximal feasible” output vector for each finite input vector, dependent on the direction in which one moves towards the production frontier. Axiom (iv) states that doing nothing is feasible, while axiom (v) states that doing something for nothing is infeasible. Convexity implies that an average of two technically feasible input-output allocations is also feasible. This axiom generalizes the concept of diminishing marginal rate of technical substitution. The axioms of free disposability secure that the production takes place in the economic region of the technology, i.e. where there is no congestion. Simply put, the axiom of free disposability of inputs implies that if an input vector can produce a certain output vector, then a larger input vector is also capable of producing that output vector. That is, free disposability of inputs generalizes the concept of positive marginal productivities of inputs. The axiom of free disposability of outputs suggests that one can always perform worse by “throwing away outputs” for any input vector.

The axiom of free disposability of outputs is problematic when a bad is incorporated among outputs. The axiom suggests that the bad output can be set equal to zero for any amounts of inputs and good outputs, since it is always possible to “do worse” in terms of the production of the bad output. This lead to the nonsensical result that zero emissions can be achieved at no costs (Førsund, 2009), i.e. without adjusting the input use and the production of good outputs. To overcome this problem, Färe et al. (1989; 2005) replace axiom (viii) by the axiom of weak disposability of good and bad outputs and introduce an additional axiom; null-jointness (Shephard and Färe, 1974). Their model comprises axioms (i)-(vii), and the additional three axioms:

- (ix) Good outputs are freely disposable,
i.e. if $(x, y, b) \in T$ and $y' \leq y$, then $(x, y', b) \in T$
- (x) Good and bad outputs are weakly disposable,

i.e. if $(x, y, b) \in T$ and $0 \leq \theta \leq 1$, then $(x, \theta y, \theta b) \in T$

(xi) Null-jointness

i.e. if $(x, y, b) \in T$ and $b=0$, then $y=0$

Axiom (x) considers proportional reductions in good and bad outputs to be feasible. It intends to capture that emission reductions are costly, since good outputs must be forgone in order to reduce the bad output. Axiom (xi) states that there cannot be any production of good outputs without some generation of the bad output.

I now turn to function representations for the technology. In the single output case, the technology may be represented by the usual “textbook” production function. In multi-output settings, distance functions are suitable primal representations of the technology. The distance functions project a point in the technology to its frontier in a given direction. The directional output distance function (Chung et al., 1997) is a popular function representation for EAPE-type technologies. It allows measuring a point’s distance from the frontier of the technology set in terms of possibilities to increase the good outputs and reduce the bad output. Consider the direction vector $\delta = (\delta_y, -\delta_b)$, where $\delta_y \in \mathfrak{R}_+^M$ and $\delta_b \in \mathfrak{R}_+$, and the directional output distance function:

$$\vec{D}_O(x, y, b; \delta_y, -\delta_b) = \sup \left\{ \beta \in \mathfrak{R} : (x, y + \beta \delta_y, b - \beta \delta_b) \in T \right\} \quad (6)$$

The directional output distance function inherits the properties of the parent technology. Under free disposability of outputs (and axioms (ix)-(x)), the distance function completely characterizes the underlying polluting technology in the sense that:

$$(x, y, b) \in T \text{ if and only if } \bar{D}_O(x, y, b; \delta_y, -\delta_b) \geq 0 \quad (7)$$

It further satisfies the translation property⁸, is homogenous of degree minus one in $(\delta_y, -\delta_b)$, non-decreasing in b , non-increasing in y , and concave in (y, b) .

3.1. Joint production of good and bad outputs revisited

Having introduced the axiomatic production model and its function representations, I now return to Hoang and Coelli's (2011) finding that the model of Färe et al. (1989; 2005) is inconsistent with the materials balance principle. Rather than evaluating the axioms underlying Färe et al.'s (1989; 2005) model, Hoang and Coelli (2011) establish an inconsistency between the materials balance principle and the desired function representation for Färe et al.'s (1989; 2005) technology; the directional output distance function. They do this by inserting the directional output distance function from equation 6 into equation 1, i.e. into the expression for uncontrolled emissions. Instead of considering a specific direction vector as in Hoang and Coelli (2011), where the direction vector is set equal to the producer's observed output of goods and bads, I follow Rødseth (2011) and derive a more general expression from equations 1 and 6:

$$(b - \beta\delta_b) = ux - v(y + \beta\delta_y) \Leftrightarrow b + vy - ux = \beta(\delta_b - v\delta_y) \quad (8)$$

The right hand side of the latter expression in equation 8 must be zero in order for the materials balance principle to hold. This condition is satisfied in two cases; 1) when the directional output distance function β is equal to zero for all producers (i.e. when all producers are technically efficient) and 2) when $\delta_b = v\delta_y$ (allowing the directional output distance function to take any non-negative value). Hoang and Coelli (2011) conclude that neither of these conditions are

desirable and that Färe et al.’s model is not suitable for cases where the materials balance principle governs the generation of the bad output.

Let us consider the meaning of the latter condition, $\delta_b = v\delta_y$, more carefully. It says that the weakly disposable production model is consistent with the materials balance principle *if* the direction vector is chosen such that the reduction in the bad output, δ_b , equals the increase in recuperation by additional production of good outputs, $v\delta_y$. This is intuitively reasonable since we consider the input vector to be fixed. Hence, if a producer becomes more efficient in terms of converting inputs into desirable outputs, say by δ_y , the recuperation of the bad output increases correspondingly by $v\delta_y$ while material inflows related to the employment of inputs are fixed. The increase in recuperation must therefore be accompanied by an equivalent decline in the bad output (δ_b) in order to maintain equality in equation 1, i.e. to maintain materials balance.

Although I have shown that an appropriate choice of the direction vector allows the directional output distance function to be in compliance with the materials balance principle, I believe that the concern should be directed towards the choice of the axioms that make up the production model. After all, they embody the properties of the technology and govern its production possibilities. To elaborate further, figure 1 presents the materials balance consistent output set for example 1 (the nutrient balance example) from table 1. For the given input vector x , the materials balance principle is represented graphically by the line with slope $-(1/v) = -(1/1.17)$, hereafter called the “materials balance line”. The three pig farms A, B, and C are all located on the materials balance line. Consider applying the model of Färe et al. (1989; 2005) to rank the performances of the producers. If the directional output distance function is estimated with direction weights selected such that $\delta_b = v\delta_y$, the projection of the good and

Figure 1

bad output to the frontier takes place along the materials balance line. Producer B is then considered to be more efficient than producers A and C since he is able to produce more of the good output and less of the bad output while consuming the same amount of inputs as A and C. Since the projection of A and C towards the frontier (to B) takes place along the materials balance line, Hoang and Coelli's criterion suggests that Färe et al.'s (1989; 2005) model is in compliance with the materials balance principle. This is, however, incorrect: While the materials balance principle suggests that all feasible production possibilities are allocated on the materials balance line, Färe et al.'s model suggests that any point in the (shaded) set bounded by OBD0 in figure 1 is feasible. Clearly, all the points to the left of the materials balance line are not consistent with the materials balance principle (more specifically, with uncontrolled emissions), although several of these points are considered to be feasible by Färe et al.'s model.

My simple graphical example illustrates that one should consider the underlying axioms that determine the reference technology. Only considering the function representation or the choice of direction vector does not provide sufficient information to evaluate the consistency of the materials balance principle and the axiomatic production model.

Hoang and Coelli's (2011) critique of Färe et al.'s (1989; 2005) model does not consider end-of-pipe abatement. In section 2, I introduced the concepts of uncontrolled and controlled emissions and showed that their difference is due to end-of-pipe abatement. Graphically, end-of-pipe abatement causes a parallel shift in the materials balance line to the left in figure 1. This implies that more of the points to the left of the depicted materials balance line, and thereby also more of output set of Färe et al.'s model, become feasible with end-of-pipe abatement.

By extending the analysis of Rødseth (2011), it can be shown that key axioms of Färe et al.'s model can be justified by end-of-pipe abatement. The axiom of weak disposability of good and bad outputs states that if a vector of good and bad outputs is feasible for given inputs, then any proportional reduction of the outputs is also feasible given the inputs. The proportional reduction is defined mathematically by multiplying the good and bad outputs with a scalar θ , where $0 \leq \theta \leq 1$; see axiom (x). By multiplying the good and bad outputs in equation 2 (the representation of controlled emissions) with the scalar θ , I derive the following expression:

$$\theta(b + vy) = ux - a \tag{9}$$

Recall that the weak disposability axiom is defined for a given input vector x , which implies that ux in equation 9 must equally be considered fixed. Thus, if there is no end-of-pipe abatement taking place (i.e. $a=0$), then the equality in equation 9 is only maintained when $\theta = 1$. In other words, the weak disposability axiom is not consistent with the materials balance principle in absence of end-of-pipe abatement. However, equation 9 holds with equality when $0 \leq \theta \leq 1$ and end-of-pipe abatement efforts increase in proportions to the reductions in the good and the bad output, by $(1-\theta)(b + vy)$. In this case, the weak disposability axiom can be defended from a materials balance perspective. Since free disposability implies weak disposability, this result also holds for axiom (viii).

The null-jointness axiom states that the production of good outputs cannot take place without some generation of the bad output. More important, the axiom implies that the good and bad outputs can simultaneously be zero for any input vector $x \in \mathfrak{R}_+^N$. This is also the case for axiom (iv), inactivity. By applying equation 2, I derive the following condition for consistency of the null-jointness- and inactivity axioms and the materials balance principle:

$$0 = ux - v0_M - a \Leftrightarrow ux = a \quad (10)$$

Equation 10 states that the axioms of null-jointness and inactivity are physically attainable *if* end-of-pipe abatement is capable of eliminating all uncontrolled emissions for any nonnegative input vector $x \in \mathfrak{R}_+^N$. For example, scrubbers installed in power plants must be capable of completely removing all uncontrolled sulfur emissions from the combustion of fossil fuels. This is a strong assumption.

Finally, axiom (vii) – free disposability of inputs – suggests “increasing returns” to end-of-pipe abatement. The axiom states that if an output vector is producible for a given input vector, then a greater input vector must be capable of producing the same output vector. Consider now two input vectors $x' \geq x$ that both are assumed to be capable of producing the output vector (y, b) . Let $a' \geq a$ be defined such that the materials balance principle is satisfied for both x' and x . By inserting the vectors into equation 2

$$\begin{aligned} b + vy &= ux - a \\ b + vy &= ux' - a' \end{aligned} \quad (11)$$

and solving with respect to $b + vy$

$$ux - a = ux' - a' \Leftrightarrow u(x' - x) = a' - a \quad (12)$$

I derive the criterion for consistency between the axiom of free disposability of inputs and the materials balance principle, namely that any increase in uncontrolled emissions due to additional input use can be offset by increases in end-of-pipe abatement efforts. One issue here is that possible increases in inputs are unbounded for given outputs by the axiom of free

disposability of inputs. Thus, if x' approaches infinity, then a' must also approach infinity to ensure that equation 12 holds with equality (i.e., to ensure materials balance). Again, this is a strong and unrealistic condition.

In conclusion, the axioms of Färe et al. (1989; 2005) may be defended by means of end-of-pipe abatement. Nevertheless, there are several problems connected to this result. First, end-of-pipe abatement is only one of several options that the producers have for reducing their emissions. A model which considers only one response to environmental regulations is unlikely to capture the producer's least costly compliance strategy (Førsund, 2009; Rødseth, 2013). Second, the model appears to be less applicable to case studies where end-of-pipe abatement is unavailable or not desired because of economic reasons (Rødseth and Romstad, 2014). Third, because the model depends on end-of-pipe abatement for physical consistency it is likely to overestimate the producers' ability to reduce their emissions. As indicated by my previous results, the axioms of weak disposability of good and bad outputs, null-jointness, inactivity, and free disposability of inputs impose strong requirements on the intensity of end-of-pipe abatement for compliance with the materials balance principle. Clearly, these requirements contradict free disposability of end-of-pipe abatement outputs, since end-of-pipe abatement must be *increased* in order to maintain materials balance in cases where the good and bad outputs are decreased or the inputs are increased. Estimated increases in end-of-pipe abatement that exceed *observed* levels of end-of-pipe abatement in the dataset are questionable and suggest that firms' possibilities to reduce emissions may be overestimated.

4. A polluting technology

The purpose of this section is to propose a new EAPE model characterized by a set of axioms that ensure consistency with the materials balance principle without resorting to *ad hoc* assumptions about emission reductions by end-of-pipe abatement. The section is divided into

three subsections. First, the axiom of G -disposability is formalized, and a weaker form of G -disposability – *weak* G -disposability – is introduced. Second, a new form of essentiality that ties the generation of the bad output to the use of pollution-generating inputs is discussed. Third, the new axioms are embedded in the neo-classical technology.

4.1. Weak G -disposability

The G -disposability axiom was introduced by Chung (1996), and is defined:

- (xii) G -disposability⁹
 i.e. if $(x, y, b) \in T$, then $(x + g_x, y - g_y, b + g_b) \in T$

G -disposability implies that inputs and outputs are disposable in the G -direction. It is a more flexible disposability assumption than weak disposability since the direction in which inputs and outputs are disposable can be chosen (for example by the researcher): any direction that involves increases in inputs and the bad output and decreases in good outputs is a potential candidate. If the technology satisfies free disposability of inputs and outputs, then it also satisfies G -disposability¹⁰, i.e. disposability in the G -direction is an option if the technology satisfies free disposability. Chung (1996) showed that the directional distance function completely characterizes the underlying technology under G -disposability, which means that equation 7 also holds under G -disposability.

A weaker form of G -disposability that includes a summing-up restriction on changes in inputs and outputs may also be considered a potential candidate for representing the possibility to dispose inputs and outputs. I dub this new axiom *weak* G -disposability:

(xiii) *weak G-disposability*

i.e. if $(x, y, b) \in T$ and $s_x g_x + s_y g_y + s_b g_b = S$

where $s_x \in \mathfrak{R}^N; s_y \in \mathfrak{R}^M; s_b \in \mathfrak{R}; S \in \mathfrak{R}_+$, then $(x + g_x, y - g_y, b + g_b) \in T$

The only difference between *weak G-disposability* and *G-disposability* is the summing-up restriction on feasible changes in inputs and outputs, i.e. $s_x g_x + s_y g_y + s_b g_b = S$. It constraints the direction in which inputs and outputs are disposable (the *G-direction*). While the *G-disposability* axiom considers disposal in any direction that involves increases in inputs and the bad output and decreases in desirable outputs to be feasible, the *weak G-disposability* axiom considers only a subset of these directions to be feasible since the weighted sum of changes in inputs and outputs must amount to the value S . Clearly, if the technology is *G-disposable*, then it is also *weakly G-disposable* since $(x + g_x, y - g_y, b + g_b) \in T$ is implied by both axioms. Hence, the directional distance function completely characterizes the underlying technology under *weak G-disposability*, following the proof of Chung (1996).

It is now straightforward to show that *weak G-disposability* allows consistency between the axiomatic production model and the materials balance principle under a certain summing-up restriction on feasible changes in inputs and outputs. Consistency does not require any changes in end-of-pipe abatement to take place. To see this, it is convenient to introduce the set of inputs and outputs that by the materials balance principle are consistent with a given level of end-of-pipe abatement, $a \in \mathfrak{R}_+$:

$$M(a) = \{(x, y, b) : ux - vy - b = a\} \quad (13)$$

Clearly, if any point $(x, y, b) \in T$ satisfies $ux - vy - b = a$, then $(x, y, b) \in M(a)$, or stated differently; $(x, y, b) \in T \cap M(a)$.

Proposition: If $(x, y, b) \in T \cap M(a)$ and T is *weakly G-disposable* with summing-up restriction $ug_x + vg_y - g_b = 0$, then $(x + g_x, y - g_y, b + g_b) \in T \cap M(a)$

Proof:

1. if $(x, y, b) \in T$ and $ug_x + vg_y - g_b = 0$, then $(x + g_x, y - g_y, b + g_b) \in T$ (weak G-disposability is satisfied since $s_x = u \in \mathfrak{R}^N, s_y = v \in \mathfrak{R}^M, s_b = -1 \in \mathfrak{R}$, and $S = 0 \in \mathfrak{R}_+$).

$$\begin{aligned}
2. \quad & ux - vy - b = a \\
& = ux - vy - b + 0 = a \\
& = ux - vy - b + (ug_x + vg_y - g_b) = a \\
& = u(x + g_x) - v(y - g_y) - (b + g_b) = a \Leftrightarrow (x + g_x, y - g_y, b + g_b) \in M(a)
\end{aligned}$$

Q.E.D.

Step 1 states that $(x + g_x, y - g_y, b + g_b) \in T$, which follows directly from the definition of *weak G-disposability*. Step 2 starts with the condition for $(x, y, b) \in M(a)$, namely that $ux - vy - b = a$. Next, I add zero to the equality, and then apply the summing-up condition $ug_x + vg_y - g_b = 0$ from the *weak G-disposability* axiom. After rewriting the equation, I find that $u(x + g_x) - v(y - g_y) - (b + g_b) = a$, which means that the point $(x + g_x, y - g_y, b + g_b)$ also belongs to the set $M(a)$. Combining steps 1 and 2, I conclude that $(x + g_x, y - g_y, b + g_b) \in T \cap M(a)$.

The proposition demonstrates that the *weak G*-disposability axiom allows for disposal of inputs and outputs in a way that is consistent with the materials balance principle. Notice that consistency does not require any changes in the intensity of end-of-pipe abatement: the abatement output is unaltered when inputs and outputs are disposed since the points (x, y, b) and $(x + g_x, y - g_y, b + g_b)$ both belong to the set $M(a)$. The *weak G*-disposability axiom thereby overcomes the problems with Färe et al.'s (1989; 2005) model that were treated in section 3.1. My approach is more in line with treating end-of-pipe abatement as a freely disposable output by assuming that a similar amount of end-of-pipe abatement is obtainable when inputs or good and bad outputs are disposed.

Another important result is that the consistency of the disposal of inputs and outputs and the materials balance principle holds in the case where no end-of-pipe abatement takes place. This follows directly from the above proposition which states that if $(x, y, b) \in T \cap M(0)$ (end-of-pipe abatement is zero), then $(x + g_x, y - g_y, b + g_b) \in T \cap M(0)$. Hence, the *weak G*-disposable technology is applicable to case studies where end-of-pipe abatement is unavailable or not adopted because of economic reasons.

Section 3 treated the disposability of inputs separately from the disposability of outputs, while the *weak G*-disposability axiom jointly considers the possibility to dispose inputs and outputs. For comparison, I now consider weak G-disposability when 1) inputs are fixed (for the output sets) and 2) outputs are fixed (for the inputs sets).

If inputs are fixed, it means that g_x is zero since no changes in (or disposal of) the inputs take place. The summing-up restriction for the (materials balance consistent) weak G-disposability axiom is then equal to $g_b = \nu g_y$. Notice that this summing-up requirement is equivalent to the criterion for consistency between the materials balance principle and the

directional output distance function from equation 8. Graphically, the *weak G*-disposability axiom implies that the good and bad outputs are disposable along the materials balance line in figure 1, where the outputs can be disposed in the direction of point D.

Let the outputs be considered fixed, i.e. $(g_y, g_b) = (0_M, 0)$. Then the summing-up restriction for the (materials balance consistent) weak G-disposability axiom is $ug_x = 0$. It is straightforward to see that the summing-up restriction holds when g_x is zero for all inputs with strictly positive material flow coefficients. This means that pollution-generating inputs are “indisposable”, i.e. it is not feasible to increase them without affecting the production of (good and) bad outputs. For inputs for which the material flow coefficients are zero, the change in inputs, g_x , can take any positive value without affecting the summing-up restriction. Such inputs are freely disposable.

4.2. Output essentiality

Section 3 discussed the axioms of inactivity and null-jointness, which both imply that good and bad outputs can simultaneously be zero for any nonnegative input vector. This assumption is at odds with the second law of thermodynamics, which rules out that the bad output can be zero for positive entries of pollution-generating inputs. This section suggests axioms that are consistent with the second law of thermodynamics.

A production process is likely to require both polluting-generating and non-polluting inputs. Recall numerical example 2 from section 2.1, where sulfur dioxide emissions from power generation relate to the sulfur contents of the fossil fuels, but not to the generating units’ consumption of capital. Consider partitioning the input vector into $1, \dots, P$ pollution-generating inputs and $P+1, \dots, N$ non-polluting inputs such that $x = (x_P, x_{NP})$, where the subscript P

denotes pollution-generating inputs and the subscript NP denotes non-polluting inputs. Define for x_P the set:

$$E(x_P) = \{x \in \mathfrak{R}_+^N : x \geq 0_N \text{ and } x_P = 0_P\} \quad (14)$$

The set $E(x_P)$ is defined such that all pollution-generating inputs are equal to zero. Using the set, I now introduce a new axiom which I call output essentiality:

- (xiv) Output essentiality (for the bad output)
 i.e. if $(x, y, b) \in T$ and $b=0$, then $x \in E(x_P)$

The output essentiality axiom states that the bad output is zero if no pollution-generating inputs are used in the production process. Positive entries of pollution-generating inputs thereby require some production of the bad output, in line with the second law of thermodynamics. Output essentiality thus rules out the inactivity axiom which states that zero bad output is feasible for any nonnegative input vector. Contrary to the inactivity axiom, the output essentiality axiom implies that output sets for nonzero vectors of pollution-generating inputs do not include the origin. In other words, the bad output is an essential (or unavoidable) output.

It is interesting to study output essentiality's relationship to the "conventional" definition of (input) essentiality, which concerns essentiality of inputs rather than outputs. Following Shephard (1970), the subset of inputs that contribute to pollution, x_P , is (input) essential to the production of bad outputs if $T \cap E(x_P)$ is empty for $b > 0$. (Input) essentiality rules out cases where the bad output is positive when no pollution-generating inputs are consumed, but it does *not* rule out cases where the consumption of pollution-generating inputs is positive and

the bad output is zero. In other words, the inactivity axiom is not contradicted by (input) essentiality. Output essentiality, on the other hand, rules out inactivity for nonzero vectors of polluting inputs. It does, however, not rule out cases where the bad output is positive while the consumption of pollution-generating inputs is zero. Consequentially, the axiomatic production model in section 4.3 includes both (input) essentiality and output essentiality to fully capture the relationship between polluting-generating inputs and the bad output. Together, (input) and output essentiality imply that the bad output cannot be produced without pollution-generating inputs, and that consumption of pollution-generating inputs leads to unavoidable pollution.

Finally, I consider output essentiality's relationship to axiom (xi), null-jointness. Assume now that pollution-generating inputs are (input) essential to the production of good outputs.

Proposition: If the bad output is (output) essential to the consumption of pollution-generating inputs and the pollution-generating inputs are (input) essential to the production of good outputs, then null-jointness (axiom (xi)) is implied.

Proof:

1. if $(x, y, b) \in T$ and $b=0$, then $x \in E(x_P)$ by output essentiality
2. if $x \in E(x_P)$, then $y = 0_M$ by (input) essentiality.

Combining step 1 and 2 of the proof, it is clear that if $(x, y, b) \in T$ and $b=0$, then $y=0$ (null-jointness) under the two forms of essentiality.

Q.E.D.

4.3. Axioms of a polluting technology

I now embed the weak G-disposability and essentiality axioms in an axiomatic production model that is consistent with the first and second laws of thermodynamics:

- (i') T is nonempty
- (ii') T is closed
- (iii') For every finite x , T is bounded from above (the *output sets* are bounded)
- (iv') Output essentiality (for the bad output)
i.e. if $(x, y, b) \in T$ and $b=0$, then $x \in E(x_P)$
- (v') (Input) essentiality (for the bad output)
i.e. if $(x, y, b) \in T$ and $b > 0$, then $T \cap E(x_P) = \phi$
- (vi') There is no free lunch, i.e. if $(y, b) \geq 0_{M+1}$ then $(0_N, y, b) \notin T$
- (vii') T is a convex set
- (viii') Inputs and outputs are *weakly* G-disposable
i.e. if $(x, y, b) \in T$ and $ug_x + vg_y - g_b = 0$, then $(x + g_x, y - g_y, b + g_b) \in T$

It is straightforward to show that axioms (i')-(iii') and (vi')-(vii') do not contradict the laws of thermodynamics. Axiom (i) states that the technology is non-empty, which means that some feasible input-output combinations exist. The materials balance principle states that whenever inputs are available they can be transformed into good outputs and byproducts, while the second law secures that the employment of pollution-generating inputs always leads to the generation of byproducts. Hence, both the axiomatic production model and the laws of thermodynamics imply that some input-output combinations are feasible when inputs are available.

Axiom (ii') states that the technology T is closed, which means that the technology set includes its boundary. Formally, a closed set can be defined by considering sequences of the input- and output vectors, (x_r, y_r, b_r) , where the limit of the sequences are $\lim_{r \rightarrow \infty} x_r = x_0$,

$\lim_{r \rightarrow \infty} y_r = y_0$, and $\lim_{r \rightarrow \infty} b_r = b_0$. If T is closed and $(x_r, y_r, b_r) \in T$ for all r , then $(x_0, y_0, b_0) \in T$

. It can equally be shown that if $(x_r, y_r, b_r) \in M(a)$ for all r , then $(x_0, y_0, b_0) \in M(a)$, where $M(a)$ is the set of inputs and outputs consistent with end-of-pipe abatement equal to a by the materials balance principle (see equation 13). Using the definition of $M(a)$ along with the summation rule for limits it follows that:

$$\begin{aligned}
& \lim_{r \rightarrow \infty} [ux_r - vy_r - b_r] = a \\
& = u \lim_{r \rightarrow \infty} x_r - v \lim_{r \rightarrow \infty} y_r - \lim_{r \rightarrow \infty} b_r = a \\
& = ux_0 - vy_0 - b_0 = a \Leftrightarrow (x_0, y_0, b_0) \in M(a)
\end{aligned} \tag{15}$$

If every input-output vector in the sequence satisfies the materials balance principle (for a given level of end-of-pipe abatement), then the limit of the sequence also satisfies the materials balance principle. Hence, whenever (x_r, y_r, b_r) is both included in the technology and satisfies the materials balance principle for all r , then the axiom of closedness is not ruled out by the materials balance principle.

Axiom (iii') states that the technology is bounded from above for each finite input vector, which means that finite amount of inputs can only produce finite amounts of outputs. When reviewing this axiom it is convenient to rewrite the materials balance principle as $vy + b + a = ux$ to consider physically feasible outputs for the finite input vector x . It follows readily from $vy + b + a = ux$ that the bad output, the abatement output, and good outputs with non-zero recuperation factors (material flow coefficients for outputs) must be finite when x is finite. If they are not, the sum $vy + b + a$ is infinite and cannot equal ux (which is assumed to be finite). However, good outputs which recuperation factors are zero can take any non-negative value without affecting the materials balance condition. Since they can both be infinite

and finite they do not rule out boundedness. In conclusion, the laws of thermodynamics do not rule out the assumption that outputs are bounded for each finite input vector.

Axiom (vi') states a positive output vector is not feasible for a nonpositive input vector, i.e. the production of good and bad outputs requires inputs. This requirement is clearly in line with the materials balance principle. Consider $vy + b + a = u0_N$, where $(y, b) > 0_{M+1}$. This is a contradiction. For positive levels of good and bad outputs, the materials balance equality cannot hold when x is the zero vector and ux is zero correspondingly, since the end-of-pipe abatement output can only be nonnegative.

Axiom (vii') states that the technology is a convex set. Mathematically, it implies that if $(x, y, b) \in T$ and $(x', y', b') \in T$, then $\lambda(x, y, b) + (1 - \lambda)(x', y', b') \in T$, $0 \leq \lambda \leq 1$. In the materials balance setting, this axiom implies that convex combinations of end-of-pipe abatement outputs are feasible. For example, if a dataset reports that two specific levels of end-of-pipe abatement are observed among the firms in a given industry, then the convexity axiom implies that convex combinations of the two levels are also feasible. To see this, let $(x, y, b) \in M(a)$ and $(x', y', b') \in M(a')$, where $a \neq a'$. It can now be shown that the convex combination $\lambda(x, y, b) + (1 - \lambda)(x', y', b')$, $0 \leq \lambda \leq 1$, belongs to the set $M(\lambda a + (1 - \lambda)a')$. Consider the two materials balance equations, $ux - vy - b = a$ and $ux' - vy' - b' = a'$, corresponding to the sets $M(a)$ and $M(a')$, respectively. (Step 1) Multiply the equations with λ and $(1 - \lambda)$, $0 \leq \lambda \leq 1$, to obtain $\lambda(ux - vy - b) = \lambda a$ and $(1 - \lambda)(ux' - vy' - b') = (1 - \lambda)a'$. (Step 2) Take the convex combination of inputs and outputs; $\lambda(ux - vy - b) + (1 - \lambda)(ux' - vy' - b')$. It follows from the equalities that were derived in step 1 that

$$\lambda(ux - vy - b) + (1 - \lambda)(ux' - vy' - b') = \lambda a + (1 - \lambda)a', \quad \text{which in turn implies that}$$

$$\lambda(x, y, b) + (1 - \lambda)(x', y', b') \in M(\lambda a + (1 - \lambda)a').$$

The assumption that convex combinations of (observed) end-of-pipe abatement efforts are feasible is clearly in line with standard economic theory and is intuitively reasonable. Notice that feasibility of taking convex combinations of end-of-pipe abatement efforts is fundamentally different from the requirements imposed on end-of-pipe abatement by Färe et al.'s (1989; 2005) model. While the convexity assumption says that the convex combination of two *observed* and thus *feasible* end-of-pipe efforts is also feasible, Färe et al.'s model possibly requires increases in end-of-pipe efforts beyond those observed in the data to be in compliance with the laws of thermodynamics.

Finally, it is easy to show that convexity must also hold for any two points (x, y, b) and (x', y', b') in $M(a)$. When both points belong to $M(a)$, then $\lambda(ux - vy - b) + (1 - \lambda)(ux' - vy' - b') = \lambda a + (1 - \lambda)a = a$. Hence, $\lambda(x, y, b) + (1 - \lambda)(x', y', b')$ belongs to $M(a)$. This means that convexity also holds for $M(0)$, i.e. in the case where no end-of-pipe abatement takes place.

5. Comparison to other models

Section 4 introduced a new EAPE model characterized by a set of axioms that are in line with the laws of thermodynamics. The purpose of this section is to apply the numerical examples from section 2.1 to compare the new production model to Coelli et al.'s (2007) MBP method and to Färe et al.'s (1989;2005) EAPE model.

5.1 Comparison to the MBP method

Coelli et al.'s (2007) model and the new production model are both consistent with the materials balance principle. However, a major difference is that the former does not introduce

pollutants *into* the production model but considers a pollution minimization problem that corresponds to the well-known cost minimization problem, using material flow coefficients instead of input prices. Formally, $\inf_x \{ux : x \in L(y)\}$.

Coelli et al. (2007) impose the traditional axioms, including free disposability, on the production model. Moreover, the contribution of good outputs to recuperate pollution is not explicitly modeled¹¹. My model, on the other hand, enforces disposability of outputs in line with the materials balance principle. Only in the case where the material flow coefficients for good outputs actually are zero, the new model assumes free disposability of good outputs. Figure 1 (the materials balance line) illustrates the relevant piecewise linear output set estimated using my production model to the data in table 1. According to the figure, any disposal of the good output is accompanied by a proportional increase in the bad output. Thus, the disposal of the good output is not free, but is costly for the environment. The new EAPE model suggests that there is a potential for farms A and C to improve their environmental efficiencies by moving along the materials balance line to the “best practice farm” (to B). Note that by moving to B, producers A and C’s revenues will increase, since they are capable of producing more of the saleable output. Thus, the new model allows pollution reduction to be profitable in some instances. Coelli et al. (2007) have also recognized this feature, which is attributed to the materials balance principle.

5.2 Comparison to the EAPE approach

This section considers the numerical example on sulfur dioxide emissions from electricity generation that was presented in section 2.1 and which table 3 summarizes. Several publications on this case study have adopted Färe et al.’s (1989; 2005) model; e.g. Färe et al. (2005; 2013).

Figure 2

Figure 2 depicts the two technologies, assuming piecewise linear variable returns to scale technologies. The bold lines indicate the boundaries of the new technology, while the dotted lines indicate the boundaries of Färe et al.'s (1989, 2005) technology. In order to avoid complex higher-dimensional figures, I present the technology in the form of three output sets, one for each of the three input vectors in table 3.

Electricity generating unit C is not involved in any end-of-pipe abatement activities. Thus, since the coal input is fixed for the output set $P(10000, 600)$, and since sulfur dioxide emissions are independent of the amount of electricity produced, the materials balance condition can only hold if 42,400 tons are emitted. Hence, $P(10000, 600)$ corresponds to the vertical line segment bounded by $C[424]$ for the new production model. $P(10000, 600)$ for Färe et al.'s model is, on the other hand, bounded by $0C[424]0$, where the line segment $0C$ is due to weak disposability. This illustrates a crucial difference between the two models. For the output set $P(10000, 600)$, no decision making units are observed reducing emissions by end-of-pipe abatement, and the new production model does therefore consider the supply of emissions to be inelastic. Färe et al.'s model, on the other hand, assumes that complete removal of emissions by end-of-pipe abatement is possible.

The output sets $P(10000, 600)$ and $P(10000, 1000)$ differ in terms of the amount of capital input under consideration. Since capital is a non-polluting input, the weak G-disposability assumption collapses into free disposability for the capital input. This means that since the generating unit C's output vector was feasible for $P(10000, 600)$, then it is also feasible for $P(10000, 1000)$. The output set $P(10000, 1000)$ for the new production technology is therefore bounded by $[85]AC[424][85]$, where the line segment AC is due to convexity. Färe et al.'s model assumes weak disposability, and its output set $P(10000, 1000)$ is thus bounded by $0AC[424]0$.

The reason why $P(10000, 600)$ differs from $P(10000, 1000)$ for the new production model, in the sense that the output set goes from being a vertical line segment to a comprehensive output set, is that the generating units A, B, and C's end-of-pipe abatement efforts differ. The depiction of $P(10000, 1000)$ illustrates that the new production model allows capturing environmental-economic trade-offs in the spirit of Färe et al. (1989; 2005), since some electricity must be forgone when moving from point A to point C in the figure. This trade-off is due to convexity, and not due to disposability axioms. Second, the new production model imposes a lower bound for sulfur dioxide emissions at 8,500 tons. This is the lowest emission level observed in the data, thus reflecting the highest end-of-pipe effort observed. It is clear from figure 2 that Färe et al.'s (1989;2005) model assumes that end-of-pipe efforts can be extended to beyond this threshold.

The third panel illustrates the output sets $P(15000, 1000)$ for the new EAPE model and Färe et al.'s (1989; 2005) model. The output set for the new model is bounded by $[300]A'DC'[636]$, while the output set for Färe et al.'s model is bounded by $0AC[424]0$. This panel illustrates an important difference between the new production model and Färe et al.'s model, namely their assumptions about input disposability. Färe et al.'s model assumes free disposability of inputs, which means that if the output vectors belonging to generating units A, B and C were feasible for $P(10000, 1000)$, then they are also feasible for $P(15000, 1000)$. The new production model, on the other hand, recognizes that the materials balance principle implies that increased coal consumption must lead to corresponding higher uncontrolled emissions of sulfur dioxide. If the generating units' end-of-pipe abatement efforts are unchanged, it means that the output set for $P(10000, 1000)$ must shift "to the right" in the figure when the coal consumption increases from 1,000,000 to 1,500,000 tons of coal and the capital expenditure (the end-of-pipe abatement input) remains constant. The virtual units A',

B' and C' correspond to the observed units A, B, and C, but their emissions are changed in proportion to the increase in coal consumption.

The difference in the possibilities to dispose inputs may have major implications for efficiency measurement. Consider measuring the efficiency of unit E by evaluating its possibility to reduce its sulfur emissions given its current inputs and electricity production. According to the new production model, the unit may reduce its emissions by moving to the virtual datapoint A'. Färe et al.'s production model, on the other hand, assumes that unit E can move to unit A. That is, instead of assuming that generating unit E's emissions can be reduced by 5,300 tons by adopting best practices, Färe et al.'s model assumes that E's emissions can be cut by 26,500 tons.

Førsund (2009) argues that the materials balance principle points to the crucial role of material inputs in pollution generation. This perspective suggests that the assumption of free disposability of polluting inputs is inappropriate for frontier-based eco-efficiency models. I believe that this point is not widely recognized in the EAPE literature.

6. Summary and conclusions

Lauwers (2009) classifies the literature on frontier-based eco-efficiency models into three categories, of which two explicitly model the relationship between inputs and outputs. One of these classes considers pollutants as inherent to the technology (EAPE models), while the other type (the MBP approach) does not introduce pollutants into the production model, but uses material flow coefficients to calculate pollution. While the EAPE models are criticized for being inconsistent with the materials balance condition (Coelli et al, 2007; Hoang and Coelli, 2011), the MBP approach is developed under the assumption that the production process under consideration does not involve any form of end-of-pipe abatement activity (Coelli et al., 2007

p. 9). In this paper, I show that the most popular EAPE model's key axioms are in line with the materials balance principle if one accepts that end-of-pipe abatement efforts readily can be adjusted to reduce emissions (even for fixed inputs). I find this assumption to be strong and unrealistic, and particularly unsuitable for applications to sectors where end-of-pipe abatement is not among the desirable abatement options. I therefore introduce a new EAPE model characterized by a set of axioms that are in line with the materials balance principle without requiring emission reductions by end-of-pipe abatement. In the paper, I show that the new production model also is applicable to case studies where actual emissions differ from uncontrolled emissions. The model can thus be seen as extending and enriching the MBP approach.

In their paper, Coelli et al. (2007) discuss how the MBP method could be extended to also account for end-of-pipe abatement. They consider including pollutants in the technology specification, but argue that this causes conceptual problems with the production model, in particular that all decision making units must be efficient. In this paper, I show that it is possible to include pollutants among the outputs in the production model, but still to allow for inefficiency under the materials balance approach. Coelli et al. (2007) also consider adding an "abatement output" (e.g., scrubbed materials) to the technology to resolve the "pollution control problem". My concern is that, while the "abatement output approach" works well in theory, it is usually difficult to obtain reliable data on end-of-pipe abatement inputs and outputs¹². In the case where data on pollutants are easily obtainable, I believe that my model offers a desirable alternative to the "abatement output approach". As illustrated by a numerical example on sulfur dioxide emissions, the model is well equipped for capturing the economics of end-of-pipe abatement.

Frontier-based eco-efficiency models are useful for applied economic analysis because of their tractability and low data requirement. In this respect, the MBP approach seems superior since it only requires data on inputs and good outputs. Lauwers (2009) points out that non-point source emission cannot be measured directly, but is usually calculated from conventional inputs and outputs using the materials balance equation. Thus, first using the materials balance principle to calculate uncontrolled emissions and thereafter to include the estimated emissions in the output vector of the new production model may seem as adding redundant information. However, as one of the examples in this paper shows, this may in fact not be the case. The reason is that the MBP approach treats good outputs as freely disposable, while my production model enforces disposal of good outputs in a way that is consistent with the materials balance condition. Only in the case where the good outputs have no material contents and there is no end-of-pipe abatement taking place, the new EAPE-production model can be seen as including redundant information relative to the MBP approach.

EAPE models have many desirable features. First, they allow using directional output distance functions to measure efficiency by evaluating the technical possibilities to simultaneously expand good outputs and contract pollutants. Second, the EAPE approach offers the possibility to estimate shadow prices for bad outputs (Färe et al., 1993). Third, the approach allows for calculating elasticities of substitution among good and bad outputs (Färe et al., 2005; 2012). The MBP approach does not explicitly model pollutants, and does therefore not readily accommodate these applications. On the other hand, the MBP approach allows one to more clearly show that pollution reduction can be cost reducing (Coelli et al., 2007) – or revenue increasing. By bridging the gap between the EAPE and MBP approaches, my model inherits these desirable features.

This paper has compared my new production model to the popular frontier-based methods of Färe et al. (1989; 2005) and Coelli et al. (2007) using simple numerical examples. My aim has been to provide examples that illustrate differences among the production models' features in a simple and transparent way. Future research should extend these comparisons using real data. Only undertaking this task will reveal how much the theoretical differences among the models really mean for applied efficiency measurement.

7. Endnotes

¹ See for example the discussion in Daly (1997) and Stiglitz (1997).

² The most common approaches are modelling pollutants as weakly disposable outputs (see Färe et al., (1989; 2005)) or freely disposable inputs (see Baumol and Oates (1975), Pittman (1981), and Barbera and McConnell (1990)).

³ Clearly, the first law of thermodynamics encompasses both material and energy balances. However, in line with the literature on polluting technologies (Coelli et al., 2007; Førsund, 2009), I focus solely on materials in the paper.

⁴ The current paper concerns one bad output (byproduct) only. The purpose is to simplify the analysis and making it more transparent. However, the analysis may be generalized to multiple bads by introducing a vector of bads along with a material flow coefficient matrix.

⁵ The material flow coefficients may vary across producers. For example, there exist various qualities of coal which differ in terms of their sulfur content and therefore require non-uniform

flow coefficients across producers. The current paper does not make any input quality assessments and does therefore not deal with this issue.

⁶See Førsund (2009) for a more detailed discussion on the dynamics of end-of-pipe abatement.

⁷See the Electric Power Annual (<http://www.eia.gov/electricity/annual/>) for information on emission factors and electricity data.

$$^8 \bar{D}_O(x, y + \alpha\delta_y, b - \alpha\delta_b; \delta_y, -\delta_b) = \bar{D}_O(x, y, b; \delta_y, -\delta_b) - \alpha, \quad \alpha \in \mathfrak{R}$$

⁹ The (selected) direction vector for the directional output distance function, δ , may differ from the G-direction, $g = (g_x, g_y, g_b)$, in which inputs and outputs are disposable according to the G-disposability axiom.

¹⁰ Notice that the G-disposability axiom is here defined such that the bad output must be considered a freely disposable *input* in order for the free disposability of inputs and outputs axioms to satisfy axiom (xii); G-disposability. The reason is that the G-disposability axiom implies that increases in inputs and the bad outputs are possible for any vector of good outputs.

¹¹ Coelli et al. (2007, p.7) correctly state that when keeping the good output vector fixed, the uncontrolled emissions (equation 1) are minimized when the aggregate material content of the inputs are minimized. Formally, $\inf_x \{ux - vy : x \in L(y)\} = \inf_x \{ux : x \in L(y)\} - vy$, where vy is a fixed discount due to recuperation. This discount does, however, not appear in Coelli et al.'s environmental efficiency measurement framework. Only when the material flow coefficients

for outputs are zero, the uncontrolled emission minimization (that includes the discount) coincides with Coelli et al.'s material inflow minimization.

¹² This point is illustrated by Hampf's (2014) and Färe et al.'s (2013) recent contributions on the environmental efficiency of U.S. power producers. These papers attempt to explicitly model end-of-pipe abatement using network technologies, but their analyses are restricted due to limited data availability.

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9. Figure legends and tables

Figure 1: *The nutrient balance example*

Figure 2: *The sulfur dioxide example*

Table I: *The nutrient balance example – data matrix*

Firm ID	Piglets	Feed	Meat	Byproduct
A	2.0	22.0	9.0	19.1
B	2.0	22.0	11.0	16.8
C	2.0	22.0	8.0	20.3

Table II: *The nutrient balance example – nutrient inflows and outflows*

Firm ID	Piglets	Feed	Meat	Byproduct
A	2.3	27.3	10.5	19.1
B	2.3	27.3	12.9	16.8
C	2.3	27.3	9.4	20.3

*Note that the columns entitled "byproduct" in Tables 1 and 2 are equivalent. Table 1 constitutes a data matrix with one desirable and one undesirable output that will be further considered later in the paper, while table 2 illustrates how the byproduct is derived by subtracting the nitrogen content of the salable meat from the sum of the nitrogen inflows due to the farms' piglet and feed consumption.

Table III: *The sulfur dioxide example – data matrix*

Firm ID	Fuel	Capital	Electricity	Uncontrolled emissions	Controlled emissions
A	10,000	1,000	250	424	85
B	10,000	1,000	200	424	127
C	10,000	600	300	424	424
D	15,000	1,000	280	636	350
E	15,000	1,000	250	636	350