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# Carbon Dioxide Emission Standards for U.S. Power Plants: An Efficiency Analysis Perspective

by

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**Abstract**

On June 25, 2013, President Obama announced his plan to introduce carbon dioxide emission standards for electricity generation. This paper proposes an efficiency analysis approach that addresses which emission rates (and standards) would be feasible if the existing generating units adopt best practices. A new efficiency measure is introduced and further decomposed to identify different sources' contributions to emission rate improvements. Estimating two Data Envelopment Analysis (DEA) models - the well-known joint production model and the new materials balance model - on a dataset consisting of 160 bituminous-fired generating units, we find that the average generating unit's electricity-to-carbon dioxide ratio is 15.3 percent below the corresponding best-practice ratio. Further examinations reveal that this discrepancy can largely be attributed to non-discretionary factors and not to managerial inefficiency. Moreover, even if the best practice ratios could be implemented, the generating units would not be able to comply with the EPA's recently proposed carbon dioxide standard.

**JEL classification**: Q53, Q48, D24

**Keywords**: Emission standards; Carbon dioxide emissions; Materials balance condition; Electricity generation; Weak G-disposability; Data Envelopment Analysis

# 1 Introduction

On June 25, 2013, President Obama announced his plan to curb U.S. carbon dioxide ($CO_2$) emissions. By sending a strong signal that the U.S. is willing to take the lead in preventing climate change President Obama put climate change at the top of the international agenda and raised hopes for a binding international agreement on climate change mitigation.

The electricity sector is the largest emitter of $CO_2$ in the U.S., and accounts for about one-third of all domestic emissions. Although regulations that curb other emissions such as sulfur dioxide ($SO_2$) and nitrogen oxides ($NO_x$) were implemented decades ago, $CO_2$ emissions from U.S. power plants are currently not constrained. On March 27, 2012, the Environmental Protection Agency (EPA) proposed a $CO_2$ emission standard of 1000 pounds of $CO_2$ per megawatt-hour for new plants, based on the performance of the natural gas combined cycle technology. In his new initiative, President Obama has directed the EPA to complete $CO_2$ emission standards for both *new* and *existing* plants.

Understanding technological capacity is important for successful environmental policies. This paper considers *feasible* emission standards for existing electricity generating units, given the current state of their technology. The paper is thereby closely linked to a recent paper by Kotchen and Mansur (2014), which analyzes how the EPA's proposed emission standard of 1000 pounds of $CO_2$ per megawatt-hour compares to the emission rates of existing and proposed electricity generating units. We extend the scope of Kotchen and Mansur's analysis by taking *efficiency improvements* into account. More specifically, we ask which emission rates would be feasible if all units operate at their technological capacity and thus, how much the electricity generating units' current emissions could be decreased if the units adopt best practices. This information is useful for designing environmental regulations that promote efficiency improvements (in the spirit of the so-called Porter hypothesis, see Porter and van der Linde (1995)). A report by the National Energy Technology Laboratory (2008) suggests that factors which are under control by the electricity generating units, e.g., operational practices and maintenance, play large roles in determining the units' efficiencies. In other words, it appears to be possible to significantly reduce $CO_2$ emissions by increasing the units' managerial efficiencies.

To identify feasible improvements in current emission rates, this paper proposes a production analysis framework for estimating the electricity generating units' maximal feasible output-to-emissions ratios. A new efficiency measure that compares the maximal feasible ratios to the generating units' actual ratios is proposed. The measure is decomposed into three components to identify the sources of improvements. We illustrate the usefulness of our approach by calculating the maximal output-to-emissions ratios and the corresponding efficiency scores for a sample of 160 bituminous-fired generating units in operation in 2011; i.e., for existing coal-fired units that will face emission standards for $CO_2$ in the future. Data Envelopment Analysis (DEA) is used to model polluting technologies, i.e. technologies that consume coal and other inputs and produce $CO_2$ emissions jointly with electricity.

The properties of polluting technologies have recently received much attention in the production analysis literature. It is now well-known that some of the "standard" (neo-classical) axioms, in particular free disposability of outputs, do not apply to pollutants (see Førsund (2009) for a detailed discussion). A popular modeling approach by Färe et al. (1989) therefore suggests to model pollutants as weakly disposable. Among others, this approach has been extensively used to estimate environmental efficiencies and marginal abatement costs for U.S. power plants (see e.g. Mekaroonreung and Johnson (2012), Färe et al. (1996, 2007b), and Coggins and Swinton (1996)). However, the Färe et al. (1989) modeling approach is criticized for not complying with physical laws, in particular with the materials balance condition (see Førsund (2009), Coelli et al. (2007) and Hoang and Coelli (2011)). This is unfortunate in our setting since the materials balance condition is highly relevant for modeling air pollutant emissions from electricity generation. Some papers have suggested modeling polluting technologies by combining the neo-classical production technology with a parametric specification of the materials balance condition to overcome the physical inconsistencies (see e.g. Rødseth (2013) and Hampf (2014)). Alternatively, the axioms of the neo-classical production model can be modified to secure consistency between the economic model and the materials balance principle. The latter approach has, to our knowledge, not been properly addressed in the literature. Recently, Rødseth (2014a) showed that a "materials balance consistent" production model can be achieved by assuming that 1) inputs and outputs are *weakly G-disposable*, and that 2) pollutants are *output essential.* Rødseth (2014a) further showed that (despite the before-mentioned critique) the model by Färe et al. (1989) is consistent with the materials balance condition under a very strong assumption, namely that reductions in pollutants take place by end-of-pipe abatement only. This is not an appropriate assumption in our case with $CO_2$ emissions from electricity generating units since end-of-pipe technologies for $CO_2$ are currently not commercialized (see Rødseth and Romstad (2014) for a discussion). We therefore find it useful to compare the results of Färe et al.'s model (hereafter, the *joint production* (JP) model) and Rødseth's model (hereafter, the *materials balance* (MB) model), to identify possible shortcomings of the well-established joint production model in settings without end-of-pipe abatement. Our paper is the first to implement the materials balance model empirically and the first to assess the differences between the two production models using real data.

Our DEA results suggest that the average generating unit's electricity-to-carbon dioxide ratio is 15.3 percent below the corresponding best-practice ratio. Unfortunately, further examinations by second-stage regressions reveal that this discrepancy can largely be attributed to contextual factors and not to managerial inefficiency. In particular, the age of the generating units has a significant impact their efficiencies. Building upon the second-stage regression results we find that the lowest feasible emission standard for the average generating unit is 1943 pounds of $CO_2$ per MWh of produced electricity, which is slightly lower than the current average emission rate of 1997 pounds per MWh produced. Consequently, the coal-fired generating units are far from being able to comply with the EPA's suggested emission standard of 1000 pounds of $CO_2$ per megawatt-hour.

Our paper is structured as follows. Section 2 describes the theoretical underpinnings of our analysis. It presents the production models and our new efficiency measure. Section 3 describes the compilation of the dataset and presents the results. Finally, section 4 concludes.

## 2 Theoretical foundations

We start by introducing the joint production and the materials balance approach to modeling environmental technologies. Building upon the nonparametric estimation of these models we introduce optimization methods to estimate the maximal ratio of a good to a bad output. This ratio is used to construct and decompose a new efficiency measure. A discussion on the bias-correction of the estimates of the efficiency measure and the use of regression techniques to identify the effect of contextual variables concludes this section.

## 2.1 Environmental production technologies

In the following discussion we focus on a production process where $m$ inputs $\boldsymbol{x} \in \mathbb{R}_+^m$ are used to produce $k$ good outputs $\boldsymbol{y} \in \mathbb{R}_+^k$. We further assume that the $m$ inputs can be split into $m_1$ polluting inputs and $m_2 = m - m_1$ non-polluting inputs, hence $\boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}^P \\ \boldsymbol{x}^{NP} \end{bmatrix}$. The consumption of polluting inputs leads to the unintended by-production of $s$ bad (or undesirable) outputs $\boldsymbol{b} \in \mathbb{R}_+^s$. The technology set $T$ of this production process is the collection of all technically feasible input-output combinations and is defined by:

$$T = \{(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) : \boldsymbol{x} \text{ can produce } (\boldsymbol{y}, \boldsymbol{b})\}. \tag{2.1}$$

Several axiomatic approaches that account for bad outputs have been proposed in the literature on microeconomic production theory (see e.g. Scheel (2001) for a survey). One of the most frequently applied models in empirical analyses is the joint production (JP) model by Färe et al. (1989). This model imposes the following axioms on the technology (see Färe and Grosskopf (2004) for further discussions):

(JP1) $T$ is nonempty.

(JP2) $T$ is closed.

(JP3) For every finite $\boldsymbol{x}$, $T$ is bounded from above.

(JP4) No free-lunch: $(\boldsymbol{0}, \boldsymbol{y}, \boldsymbol{b}) \notin T$ if $(\boldsymbol{y}, \boldsymbol{b}) \geq (\boldsymbol{0}, \boldsymbol{0})$.[1]

(JP5) Convexity:
If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $\left(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{b}}\right) \in T$, then $\alpha (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) + (1 - \alpha) \left(\tilde{\boldsymbol{x}}, \tilde{\boldsymbol{y}}, \tilde{\boldsymbol{b}}\right) \in T$ with $\alpha \in [0, 1]$.

(JP6) Inactivity: $(\boldsymbol{x}, \boldsymbol{0}, \boldsymbol{0}) \in T$.

(JP7) Strong disposability of inputs: If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $\tilde{\boldsymbol{x}} \geqq \boldsymbol{x}$, then $(\tilde{\boldsymbol{x}}, \boldsymbol{y}, \boldsymbol{b}) \in T$.

(JP8) Strong disposability of good outputs:
If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $\tilde{\boldsymbol{y}} \leqq \boldsymbol{y}$, then $(\boldsymbol{x}, \tilde{\boldsymbol{y}}, \boldsymbol{b}) \in T$.

(JP9) Weak disposability of good and bad outputs:
If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$, then $(\boldsymbol{x}, \theta\boldsymbol{y}, \theta\boldsymbol{b}) \in T$ with $0 \leqq \theta \leqq 1$.

(JP10) Null-jointness: If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $\boldsymbol{b} = \boldsymbol{0}$, then $\boldsymbol{y} = \boldsymbol{0}$.

The main differences between the conventional technology set that does not account for the production of pollutants (see Shephard (1970) for an overview) and the joint production model are the axioms (JP9) and (JP10). The weak disposability axiom (JP9) states that a reduction in the bad outputs is costly since the production of good outputs must be reduced correspondingly, i.e. revenues must be forgone. The rationale behind this assumption is that inputs are reallocated from the production of the good outputs to the abatement of the bad outputs. The null-jointness assumption (JP10) states

[1] Following the usual notational convention we use $\geq$ and $\leq$ if at least one element of a vector satisfies strict inequality while $\geqq$ and $\leqq$ imply that each element can hold with equality.

that no good outputs can be produced without some by-production of bad outputs. In the words of Färe et al. (2007a, p. 1057), "there is no fire without smoke".

While the joint production model provides a theoretically appealing approach to incorporate pollutants as bad outputs, it is in general not able to account for the laws of thermodynamics (see Coelli et al. (2007)). The literature on environmental economics (see e.g. Baumgärtner et al. (2001)) highlights in particular the role of the first and second laws of thermodynamics in determining pollution from conventional production processes. In line with previous studies (see Coelli et al. (2007)) we limit our discussion on the first law of thermodynamics to the materials balance condition (MCB). The MBC, which was introduced in the economic literature by Ayers and Kneese (1969), states that the amount of materials bound in the inputs must be equal to the amount of materials bound in the intended outputs and the production residuals, which in our case translates to the good and bad outputs.[2] Given our above presented production process the MBC reads as equation (2.2)

$$\boldsymbol{S_x x} = \boldsymbol{S_y y} + \boldsymbol{b} + \boldsymbol{a} \tag{2.2}$$

where $\boldsymbol{S_x}$ denotes the $s \times m$ matrix which indicates the amount of materials bound in the inputs (i.e., emission factors). Since the non-polluting inputs do not contain any materials the last $m_2$ rows of the matrix do only contain zeros. $\boldsymbol{S_y}$ denotes the $s \times k$ matrix which indicates the amount of materials bound in the good outputs (i.e., recuperation factors), and $\boldsymbol{a}$ represents a $s \times 1$ vector containing the amount of abatement for each pollutant.[3] In this definition of the MBC the amount of materials bound in the inputs corresponds to the sum of the materials bound in the good outputs, the amount of bad outputs, and the amount of abatement. In our empirical case study the matrix $\boldsymbol{S_y}$ is the zero matrix since the good output (electricity) does not contain any materials. Moreover, $\boldsymbol{a} = \boldsymbol{0}$ since no abatement activities for carbon dioxide are present.[4]

While the MBC states that materials cannot vanish during the production process, the second law of thermodynamics states that polluting inputs cannot be completely transformed into good outputs. Therefore, the bad outputs must be strictly positive if a strictly positive amount of the polluting inputs is used (see Ebert and Welsch (2007)) if no abatement activities are present. If abatement takes place ($\boldsymbol{a} > \boldsymbol{0}$), then in theory the bad outputs might be zero when a strictly positive amount of polluting inputs is consumed. However, the case of complete removal of the bad outputs in equation (2.2) is rarely observed.

As described in the introduction to this paper, the joint production model is only consistent with the materials balance condition if (end-of-pipe) abatement possibilities are present and can be adjusted

[2] In the economy-wide perspective, all material inflows to economic processes will return to the environment. However, the materials balance principle is additive, and the fundamental materials balance equation can easily be transposed to narrowly defined systems, where inputs are intermediates instead of raw materials and where good outputs go through a transformation chain before ending up as residual materials from the consumer sector (Lauwers 2009, p. 1606). Narrow system delimitation will inevitably impose rigorous consideration of the materials balance condition (Lauwers 2009, p. 1611).

[3] The term abatement is frequently used for all types of emission reducing efforts, including input substitution and reductions in the scale of operations. In our setting, abatement primarily refers to end-of-pipe abatement, but may also encompass various forms of change-in-process abatement. See Rødseth and Romstad (2014) for a detailed discussion.

[4] In the following theoretical discussions we will assume that $\boldsymbol{a} = \boldsymbol{0}$. This is done for notational easiness in the following formal derivations of the programming problems. However, note that these derivations can be easily adapted accounting for $\boldsymbol{a} > \boldsymbol{0}$.

such that the materials balance condition is satisfied when inputs and good and bad outputs are disposed. Rødseth (2014b) rewrote the MBC (Eq. 2.2) and multiplied the outputs with the scalar $\theta$, i.e., $\boldsymbol{S_x x} - \boldsymbol{a} = \theta\left(\boldsymbol{S_y y} + \boldsymbol{b}\right)$, to show that the weak disposability axiom can only be consistent with the materials balance condition if the abatement output can be increased proportionally to the reduction in good and bad outputs by $\theta$ for a given input vector. Moreover, the null-jointness and inactivity axioms of the JP model violate the second law of thermodynamics. To overcome these drawbacks Rødseth (2014a) proposed a production model that is in line with both laws of thermodynamics. In Rødseth's paper the G-disposability axiom proposed by Chung (1997) is extended by a summing-up condition to allow weak G-disposability, which can be defined to ensure that the production model satisfies the MBC. Moreover, the concepts of input and output essentiality for the bad outputs are introduced to provide a model which does not violate the second law of thermodynamics. The full set of axioms of the materials balance (MB) approach reads as (see Rødseth (2014a) for a complete discussion)

(MB1) $T$ is nonempty.

(MB2) $T$ is closed.

(MB3) For every finite $\boldsymbol{x}$, $T$ is bounded from above.

(MB4) Output essentiality for the bad outputs: If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $\boldsymbol{b} = \boldsymbol{0}$, then $\boldsymbol{x}^P = \boldsymbol{0}$.

(MB5) Input essentiality for the bad outputs: If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $\boldsymbol{x}^P = \boldsymbol{0}$, then $\boldsymbol{b} = \boldsymbol{0}$.

(MB6) No free-lunch.

(MB7) $T$ is convex.

(MB8) Inputs and outputs are weakly G-disposable:
If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) \in T$ and $\boldsymbol{S_x g_x} + \boldsymbol{S_y g_y} - \boldsymbol{g_b} = \boldsymbol{0}$, then $\left(\boldsymbol{x} + \boldsymbol{g_x}, \boldsymbol{y} - \boldsymbol{g_y}, \boldsymbol{b} + \boldsymbol{g_b}\right) \in T$.[5]

Axioms (MB4) and (MB5) ensure that the second law of thermodynamics is not violated by stating that it is not possible to completely transform polluting inputs into good outputs. The summing-up condition in (MB8) states that the increases in pollution due to increases in the use of inputs $(\boldsymbol{S_x g_x})$ and/or the reduction of good outputs $\left(\boldsymbol{S_y g_y}\right)$ must equal the increases in the bad outputs $(\boldsymbol{g_b})$ when inputs and outputs are disposed. Hence, the MBC is satisfied. Since we assume zero abatement we also assume zero changes $(\boldsymbol{g_a} = \boldsymbol{0})$ in abatement. If this is not the case, (MB8) could be modified to: If $(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{a}, \boldsymbol{b}) \in T$ and $\boldsymbol{S_x g_x} + \boldsymbol{S_y g_y} + \boldsymbol{g_a} - \boldsymbol{g_b} = \boldsymbol{0}$, then $\left(\boldsymbol{x} + \boldsymbol{g_x}, \boldsymbol{y} - \boldsymbol{g_y}, \boldsymbol{a} - \boldsymbol{g_a}, \boldsymbol{b} + \boldsymbol{g_b}\right) \in T$ to allow for disposal of the abatement output.

In the following we present a simple graphical example to demonstrate the differences between the joint production model and the materials balance model. We consider a case with one polluting input ($x$) and two outputs, one good ($y$) and one bad ($b$). Moreover, we assume that the emission factor for the polluting input is 0.4 and that the recuperation factor for the good output is zero. In this setting, we construct a dataset containing three ($l = A, B, C$) DMUs which is presented in table I.

Using this graphical example we want to highlight how the different assumptions regarding the disposability of inputs and outputs influence the production possibilities, i.e. the output sets. To illustrate

[5] $\boldsymbol{g_x}, \boldsymbol{g_y}$ and $\boldsymbol{g_b}$ are directional vectors which model changes in the inputs and outputs that satisfy equation (2.2). Hence, they determine the direction in which inputs and outputs are disposable. The summing-up condition in (MB8) constraints the choice of directions, hence the term weak G-disposability.

Table I: Artificial data

| Firm ID | $x^l$ | $y^l$ | $b^l$ |
|---|---|---|---|
| $A$ | 10 | 4.5 | 2 |
| $B$ | 10 | 5 | 4 |
| $C$ | 20 | 5 | 8 |

the difference with regard to the disposability of outputs we compare the output sets for a fixed level of polluting inputs ($x = 10$). The implications of the differences in the disposability of polluting inputs are discussed by comparing the output sets for two levels of polluting inputs ($x = 10$ and $x = 20$). We assume a variable returns to scale technology to highlight the role of the disposability axioms in estimating production possibilities. Under constant returns to scale the output sets for the technology $T$, $P(x)$, satisfy the condition $P(tx) = tP(x), t > 0$ which leads to a more complex figure as will be described in more detail below.

Figure 1 presents the output sets for both models (joint production and materials balance) and two different input levels ($x = 10$ and $x = 20$). Note that the figure thereby deviates from most graphical representations in the literature on polluting technologies that usually depict one output set given a fixed input vector.

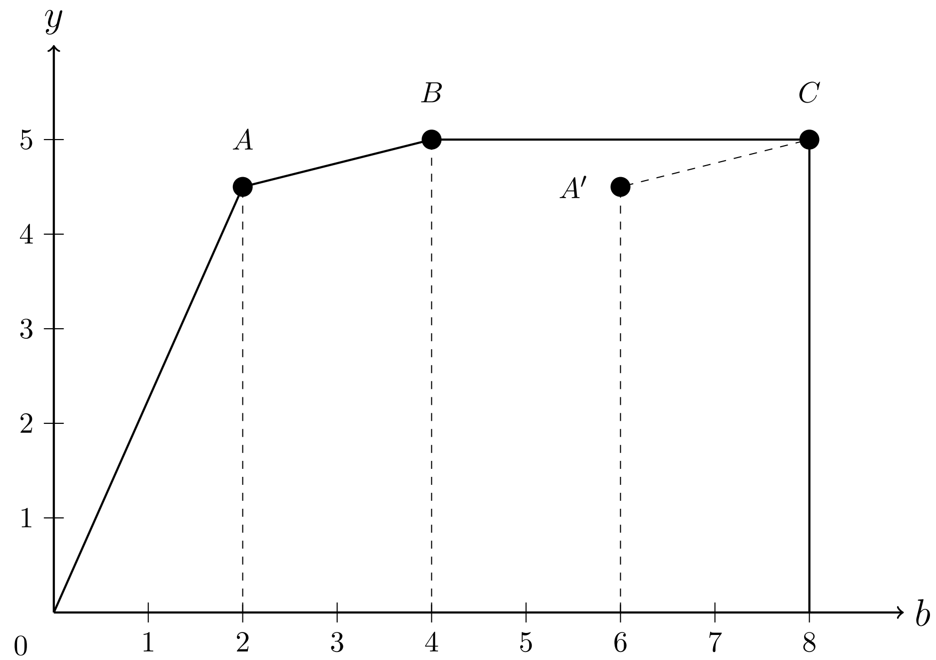


Figure 1: Output sets of the joint production and the materials balance model

Since the DMUs $A$ and $B$ both consume 10 units of the input, the output sets for $x = 10$ are estimated on the basis of these two observations. The output set for the materials balance model is bounded by $2AB42$, where the vertical line segments $2A$ and $B4$ are due to weak G-disposability of the good output. Since we assume that the recuperation factor of $y$ is equal to zero ($s_y = 0$), the amount of $y$ can be freely reduced for fixed levels of $x$ and $b$ without violating the materials balance condition (2.2). The line segment $AB$ follows from the assumption of a convex technology which implies convex output sets. From this output set it is obvious that for the materials balance model the minimal amount of $b$ given $x = 10$ is 2.

In contrast to the materials balance model which is based on the assumption of weak G-disposability, the joint production model assumes strong disposability of inputs and good outputs and weak disposability of bad outputs. Without any disposability of the bad outputs, the output set for the JP model would be the same as the output set of the MB model due to the assumption of strong disposability of good outputs. However, the weak disposability assumption states that if $(y, b) \in P(10)$, then $(\theta y, \theta b) \in P(10)$ with $0 \leqq \theta \leqq 1$, implying that $(0, 0) \in P(10)$ is feasible. This assumption extends the output set by the area bounded by the line segment $0A$. Hence, the output set of the joint production model for $x = 10$ is bounded by $0AB40$ and the minimal amount of $b$ given $x = 10$ is 0.

By the above discussion we have demonstrated how the different assumptions about the possibilities to dispose outputs shape the output sets and, thus, feasible output combinations given $x = 10$. To show how different assumptions regarding the disposal of polluting inputs affect the output sets, we include observation $C$ which uses $x^C = 20$ to produce $y^C = 5$ and $b^C = 8$.

The assumption of strong disposability of polluting inputs of the joint production model implies that if $(y, b) \in P(10)$, then $(y, b) \in P(20)$. Therefore, the output set for the joint production model and $x = 20$ contains all point of the output set for $x = 10$ and all points located on (due to convexity) or below (due to free disposability of good outputs) the line segment $BC$ (i.e., by combining the output set for $x = 10$ with observation $C$). Hence, the output set is bounded by $0ABC80$ and the minimal amount of $b$ given $x = 20$ remains 0 for the JP model.

In contrast, the MB model assumes weak G-disposability of inputs and outputs implying that if $(y, b) \in P(10)$, then $(y, b + 0.4 \cdot 10) \in P(10 + 10)$ where $s_x = 0.4$ and $g_x = 10$. Pictorially, this means that the output set of the MB model for $x = 10$ moves "to the right" when the input consumption increases from 10 to 20. Hence, in contrast to the JP model the output combinations for $x = 10$ are not technically feasible for $x = 20$. For example, the artificial observation $A'$ corresponds to DMU $A$ for $x = 20$, since at this artificial observation the good output is the same as for DMU $A$ and the bad output is increased to $b + 0.4 \cdot 10 = 4 + 4 = 8$. Hence, the output set of the materials balance model for $x = 20$ amounts to the set bounded by $6A'C86$, and the minimal feasible amount of the bad output is 6 for $x = 20$. Thus, it is larger than the minimal amount of 2 for $x = 20$, highlighting the difference in the output sets for the MB and the JP model due to different disposability assumptions. In essence, the MB model assumes a closer relationship between the consumption of polluting inputs and emissions than the JP model which assumes that inputs are freely disposable.

Note that for this graphical discussion we have assumed a variable returns to scale technology. Assuming constant returns to scale, the output sets for both models for $x = 20$ would also contain the artificial points $P\left(2 \cdot x^A\right) = 2 \cdot \left(y^A, b^A\right) = (9, 4)$ and $P\left(2 \cdot x^B\right) = 2 \cdot \left(y^B, b^B\right) = (10, 8)$ which implies an expansion of the output sets. Since this would lead to a less clear presentation of the effects of the disposability assumptions, we restrict our presentation to the case of variable returns to scale which does not allow for this expansion.

## 2.2 Nonparametric estimation and ratio optimization

We apply nonparametric methods (Data Envelopment Analysis (DEA) proposed by Charnes et al. (1978)) which do not rely on a specific functional form of the production function to estimate the technologies. Given a sample of $n$ decision making units (DMUs) with the observed input-output combinations $(\boldsymbol{x}_i, \boldsymbol{y}_i, \boldsymbol{b}_i)$ with $i = 1, \ldots, n$ the estimation of the joint production model assuming variable returns to scale (VRS) reads as (see Färe and Grosskopf (2003))

$$\widehat{T}_{JP} = \left\{(\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) : \boldsymbol{x} \geqq \boldsymbol{X}\boldsymbol{\lambda}, \boldsymbol{y} \leqq \boldsymbol{Y}\boldsymbol{\lambda}\theta, \boldsymbol{b} = \boldsymbol{B}\boldsymbol{\lambda}\theta, \mathbf{1}^T\boldsymbol{\lambda} = 1, 0 \leqq \theta \leqq 1, \boldsymbol{\lambda} \geqq \mathbf{0}\right\}. \tag{2.3}$$

In this formulation $\boldsymbol{X}$ denotes the $m \times n$ matrix of (polluting and non-polluting) inputs, $\boldsymbol{Y}$ denotes the $k \times n$ matrix of good outputs and $\boldsymbol{B}$ denotes the $s \times n$ matrix of bad outputs. $\boldsymbol{\lambda}$ represents a $n \times 1$ vector of weight factors while $\theta$ denotes the scaling factor of the weak disposability assumption (JP9). The inequality constraints on the inputs and the good outputs imply strong disposability while the equality constraints on the bad outputs impose weak disposability. The technology exhibits null-jointness of the good and the bad outputs if each DMU produces a strictly positive amount of at least one bad output and each bad output is produced by at least a single DMU (see e.g. Färe (2010)) . This technology can be modified to exhibit constant returns to scale (CRS) by removing the summing up condition on the weight factors $\mathbf{1}^T\boldsymbol{\lambda} = 1$. In this case the scaling factor $\theta$ can be set equal to one (see Färe and Grosskopf (2003)).

The corresponding nonparametric estimation of the materials balance model (MB) reads as

$$\begin{aligned}\widehat{T}_{MB} = \big\{ (\boldsymbol{x}, \boldsymbol{y}, \boldsymbol{b}) : \boldsymbol{x} = \boldsymbol{X}\boldsymbol{\lambda} + \boldsymbol{\epsilon_x}, \boldsymbol{y} = \boldsymbol{Y}\boldsymbol{\lambda} - \boldsymbol{\epsilon_y}, \boldsymbol{b} = \boldsymbol{B}\boldsymbol{\lambda} + \boldsymbol{\epsilon_b}, \\ \boldsymbol{S_x}\boldsymbol{\epsilon_x} + \boldsymbol{S_y}\boldsymbol{\epsilon_y} = \boldsymbol{\epsilon_b}, \mathbf{1}^T\boldsymbol{\lambda} = 1, \boldsymbol{\lambda} \geqq \mathbf{0}\big\}.\end{aligned} \tag{2.4}$$

In this estimation we have replaced the unspecified directional vectors of the weak G-disposability assumption (MB8) by the specific slacks in the inputs and outputs $\boldsymbol{\epsilon_x}$, $\boldsymbol{\epsilon_y}$ and $\boldsymbol{\epsilon_b}$. Hence, production points that are neither observations in the sample nor convex combinations of the observations are only part of the technology if their slacks satisfy the summing-up constraint $\boldsymbol{S_x}\boldsymbol{\epsilon_x} + \boldsymbol{S_y}\boldsymbol{\epsilon_y} = \boldsymbol{\epsilon_b}$, hence the materials balance condition. Note that other specifications of the directional vectors are also possible, e.g. for evaluating substitution among polluting inputs for fixed levels of $\boldsymbol{b}$ ($\boldsymbol{g_b} = \mathbf{0}$). The technology exhibits output essentiality for the bad outputs (MB4) if each DMU uses a strictly positive amount of the polluting inputs and produces a strictly positive amount of the bad outputs.

Based on estimates of the two technologies, the purpose of this paper is to undertake an efficiency analysis of U.S. power plants to, among others, examine whether the existing plants would be able to comply with EPA's proposed carbon standard by adopting best-practices. In most empirical studies distance functions are applied to evaluate the efficiency of power plants. For example, Barros and Peypoch (2008) use an output distance function to estimate the efficiency and analyze the effect of pollutants on the efficiency by using second-stage regressions. Other studies (e.g. Färe et al. (2007a)) apply directional distance functions (DDF) when including pollutants as weakly disposable outputs (see Zhang and Choi (2014) for a survey on the use of directional distance functions in efficiency studies in the energy sector). However, the use of DDFs in combination with weakly disposable outputs is criticized by Aparicio et al. (2013) and Chen (2013). They point out that the model by Färe et al.

(1989) may lead to negatively sloped regions of the frontiers of the output sets, and that points located on these parts of the frontier can be misclassified as efficient when applying DDFs. Sueyoshi and Goto (2012a, 2012b) propose non-radial slacks-based measures which do not suffer from this drawback of the analyses using DDFs to evaluate the efficiency of power plants. To estimate the feasibility of the EPA proposal we also propose a non-radial approach that is based on finding the optimal ratio of $y/b$ subject to the nonparametric technology sets defined above.[6]

Our approach to estimating optimal ratios is similar to the approaches of Färe et al. (2004) and Kuosmanen and Kortelainen (2005). Färe et al. (2004) propose an index of good to bad outputs which is based on ratios of radial distance functions leading to the same difficulties as the application of directional distance functions discussed above. Kuosmanen and Kortelainen (2005) present an analysis that is based on the ratio of value added to a weighted sum of environmental pressures. The weights of the environmental pressures are determined by a multiplier form of DEA, hence the model treats the pressures as inputs and keeps the output (the value added) constant. In contrast to the previous literature, we show how an optimal ratio of good to bad outputs can be estimated non-radially without using distance functions, by simultaneously optimizing $y$ and $b$.

In the following we present the optimization problems to estimate the optimal ratio $y/b$ given the specification applied for our empirical analysis. Hence, we assume scalar polluting and non-polluting inputs as well as scalar good and bad outputs. The corresponding programming problems for a general specification including multiple inputs and outputs can be found in appendix B. We start by discussing the optimization for the joint production model under constant returns to scale.

$$\begin{array}{lrcl} \max\limits_{y,b,\boldsymbol{\lambda}} & \frac{y}{b} & & \\ \text{s.t.} & x_i^P & \geqq & \boldsymbol{x}^{P^T}\boldsymbol{\lambda} \\ & x_i^{NP} & \geqq & \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} \\ & y & \leqq & \boldsymbol{y}^T\boldsymbol{\lambda} \\ & b & = & \boldsymbol{b}^T\boldsymbol{\lambda} \\ & y,b & \geqq & 0 \\ & \boldsymbol{\lambda} & \geqq & \mathbf{0}. \end{array} \quad (2.5) \quad \xrightarrow{\text{Linearization}} \quad \begin{array}{lrcl} \max\limits_{\boldsymbol{z},w} & \boldsymbol{y}^T\boldsymbol{z} & & \\ \text{s.t.} & wx_i^P & \geqq & \boldsymbol{x}^{P^T}\boldsymbol{z} \\ & wx_i^{NP} & \geqq & \boldsymbol{x}^{NP^T}\boldsymbol{z} \\ & \boldsymbol{b}^T\boldsymbol{z} & = & 1 \\ & w & \geqq & 0 \\ & \boldsymbol{z} & \geqq & \mathbf{0}. \end{array} \quad (2.6)$$

In these programming problems we have separated the constraints on the inputs. $\boldsymbol{x}^{P^T}$ ($\boldsymbol{x}^{NP^T}$) denotes the transpose of the $n \times 1$ vector of polluting (non-polluting) inputs. Equation (2.5) presents the non-linear optimization problem for the estimation of the optimal ratio for $y/b$ while equation (2.6) presents the linearized version of this optimization. Here, $\boldsymbol{z} = \frac{1}{\boldsymbol{b}^T\boldsymbol{\lambda}}\boldsymbol{\lambda}$ and $w = \frac{1}{\boldsymbol{b}^T\boldsymbol{\lambda}}$ where $\boldsymbol{z}$ can be interpreted as the "virtual" weights of the reference DMUs similar to the "virtual" multipliers in the (dual) multiplier version of DEA. A detailed discussion on the linearization of this and the following programming problems can be found in appendix A. For further discussions on linearization of non-linear DEA models see Zhou et al. (2008).

Assuming a joint production model under variable returns to scale, the programming problems read as

[6] Note that instead of minimizing the ratio $b/y$ for the estimated technology sets we maximize the ratio $y/b$. This is done to make our results and our ratio efficiency measure comparable to other approaches in the efficiency analysis literature. Since this inversion does not change the optimal results for the reference observations the policy implications of our empirical analysis are not influenced by it.

$$
\begin{aligned}
\max_{y,b,\boldsymbol{\lambda},\theta} \quad & \frac{y}{b} \\
\text{s.t.} \quad x_i^P &\geqq \boldsymbol{x}^{P^T}\boldsymbol{\lambda} \\
x_i^{NP} &\geqq \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} \\
y &\leqq \boldsymbol{y}^T\boldsymbol{\lambda}\theta \\
b &= \boldsymbol{b}^T\boldsymbol{\lambda}\theta \\
\mathbf{1}^T\boldsymbol{\lambda} &= 1 \\
0 \leqq &\ \theta \leqq 1 \\
y, b &\geqq 0 \\
\boldsymbol{\lambda} &\geqq \mathbf{0}.
\end{aligned} \tag{2.7}
\quad \xrightarrow{\text{Linearization}} \quad
\begin{aligned}
\max_{\boldsymbol{g},h} \quad & \boldsymbol{y}^T\boldsymbol{g} \\
\text{s.t.} \quad \left(\boldsymbol{x}^P - x_i^P\right)^T\boldsymbol{g} &\leqq 0 \\
\left(\boldsymbol{x}^{NP} - x_i^{NP}\right)^T\boldsymbol{g} &\leqq 0 \\
\mathbf{1}^T\boldsymbol{g} &\leqq h \\
\boldsymbol{b}^T\boldsymbol{g} &= 1 \\
h &\geqq 0 \\
\boldsymbol{g} &\geqq \mathbf{0}.
\end{aligned} \tag{2.8}
$$

In addition to the variables defined for the analysis under CRS the weak disposability factor $\theta$ has to be determined endogenously for each DMU. Moreover, the sum of the $\lambda$-values is restricted to be equal to one. Again, the non-linear programming problem (2.7) can be linearized to problem (2.8). In this programing problem $\boldsymbol{g} = \frac{1}{\boldsymbol{b}^T\boldsymbol{\lambda}\theta}\boldsymbol{\lambda}\theta$ and $h = \frac{1}{\boldsymbol{b}^T\boldsymbol{\lambda}\theta}$.

It is also possible to estimate the optimal ratio $y/b$ based on the materials balance model. The corresponding optimization problems under constant returns to scale read as

$$
\begin{aligned}
\max_{\substack{y,b,\epsilon_{x^P},\\ \epsilon_{x^{NP}},\epsilon_y,\epsilon_b,\boldsymbol{\lambda}}} \quad & \frac{y}{b} \\
\text{s.t.} \quad x_i^P &= \boldsymbol{x}^{P^T}\boldsymbol{\lambda} + \epsilon_{x^P} \\
x_i^{NP} &= \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} + \epsilon_{x^{NP}} \\
y &= \boldsymbol{y}^T\boldsymbol{\lambda} - \epsilon_y \\
b &= \boldsymbol{b}^T\boldsymbol{\lambda} + \epsilon_b \\
\epsilon_b &= s_{x^P}\epsilon_x^P + s_y\epsilon_y \\
y, b &\geqq 0 \\
\epsilon_{x^P}, \epsilon_{x^{NP}}, \epsilon_y, \epsilon_b &\geqq 0 \\
\boldsymbol{\lambda} &\geqq \mathbf{0}.
\end{aligned} \tag{2.9}
\quad \xrightarrow{\text{Linearization}} \quad
\begin{aligned}
\max_{\boldsymbol{c},v} \quad & \boldsymbol{y}^T\boldsymbol{c} \\
\text{s.t.} \quad v x_i^{NP} &\geqq \boldsymbol{x}^{NP^T}\boldsymbol{c} \\
\boldsymbol{b}^T\boldsymbol{c} + s_x x_i^P v &= 1 \\
v &\geqq 0 \\
\boldsymbol{c} &\geqq \mathbf{0}.
\end{aligned} \tag{2.10}
$$

In addition to the variables defined for the materials balance technology as well as for the optimization problems presented above, $\epsilon_{x^P}$ ($\epsilon_{x^{NP}}$) denotes the slack in the polluting (non-polluting) input and $s_{x^P}$ denotes the emission factor for the polluting input. In the linearized model $\boldsymbol{c} = \frac{1}{\left(\boldsymbol{b} - s_{x^P}\boldsymbol{x}^P\right)^T\boldsymbol{\lambda} + s_{x^P}x_i^P}\boldsymbol{\lambda}$ and $v = \frac{1}{\left(\boldsymbol{b} - s_{x^P}\boldsymbol{x}^P\right)^T\boldsymbol{\lambda} + s_{x^P}x_i^P}$.

Finally, the programming problems for the MB model under variable returns to scale are given by

$$
\begin{aligned}
\max_{\substack{y, b, \epsilon_{x^P}, \\ \epsilon_{x^{NP}}, \epsilon_y, \epsilon_b, \boldsymbol{\lambda}}} \quad & \frac{y}{b} \\
\text{s.t. } x_i^P &= \boldsymbol{x}^{P^T}\boldsymbol{\lambda} + \epsilon_{x^P} \\
x_i^{NP} &= \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} + \epsilon_{x^{NP}} \\
y &= \boldsymbol{y}^T\boldsymbol{\lambda} - \epsilon_y \\
b &= \boldsymbol{b}^T\boldsymbol{\lambda} + \epsilon_b \\
\boldsymbol{1}^T\boldsymbol{\lambda} &= 1 \\
\epsilon_b &= s_{x^P}\epsilon_x^P + s_y\epsilon_y \\
y, b &\geqq 0 \\
\epsilon_{x^P}, \epsilon_{x^{NP}}, \epsilon_y, \epsilon_b &\geqq 0 \\
\boldsymbol{\lambda} &\geqq \boldsymbol{0}.
\end{aligned}
\qquad (2.11) \qquad \xRightarrow{\text{Linearization}} \qquad
\begin{aligned}
\max_{\boldsymbol{c}, v} \quad & \boldsymbol{y}^T\boldsymbol{c} \\
\text{s.t. } v x_i^{NP} &\geqq \boldsymbol{x}^{NP^T}\boldsymbol{c} \\
\boldsymbol{1}^T\boldsymbol{c} &= v \\
\boldsymbol{b}^T\boldsymbol{c} + s_x x_i^P v &= 1 \\
v &\geqq 0 \\
\boldsymbol{c} &\geqq \boldsymbol{0}.
\end{aligned}
\qquad (2.12)
$$

In the above presented optimization models we have made two implicit assumptions. First, we have assumed that the inputs are exogenously given and cannot be adjusted by the DMUs. Second, we have assumed that $y$ can be freely chosen by the DMUs. In this case we denote the solutions to the above programming problems $r^*_{Ex} = \frac{y^*_{Ex}}{b^*_{Ex}}$. If the DMUs can adjust the amount of the inputs, $\boldsymbol{x}$ becomes an additional variable to be endogenously determined by the programming problems. We denote the solutions to these modified programming problems $r^*_{En} = \frac{y^*_{En}}{b^*_{En}}$. In addition we also account for the situation where the inputs are fixed and the DMUs cannot freely choose $y$. We consider the most relevant case that the DMUs cannot decrease the good outputs (e.g. the production of electricity) to further improve the optimal ratio. To model this case we include an additional constraint which prevents the optimal amount of $y$ from being smaller than the actual observed amount for each DMU. We denote the optimal ratio obtained by this analysis by $r^C_{Ex} = \frac{y^C_{Ex}}{b^C_{Ex}}$, with the superscript "C" indicating that the additional constraint on the good output is included when optimizing the ratio.

Given the different degrees of freedom of the DMUs to adjust the inputs as well as the good output we obtain the following relationship among the above defined ratios

$$r^*_{En} \geqq r^*_{Ex} \geqq r^C_{Ex}. \qquad (2.13)$$

## 2.3 A ratio efficiency measure

The results obtained by the above presented programs enable us to analyze the best feasible ratios of good to bad outputs for each DMU (i.e. for each electricity generating unit in our empirical application). However, these results do not provide information on how efficient the DMUs are in achieving the optimal ratios. To compare their actual performances to best practices we propose a ratio efficiency measure (REM) given by the ratio of the estimated optimal ratio to the actual observed ratio $r_{act}$ in the case with endogenous inputs and without output constraint. This implies that the inputs and outputs can be freely adjusted, hence increased or decreased, without any restrictions except the technological constraints. Therefore, the REM is defined as:

$$\text{REM} = \frac{r^*_{En}}{r_{act}} \qquad (2.14)$$

A DMU is classified as efficient (inefficient) if the measure exhibits a value equal to (larger than) one. The potential percentage increase in the ratio of good to bad outputs by adopting best-practice technology can thus be calculated by $100 \cdot (\text{REM} - 1)$.

The REM can be calculated for the joint production model and for the materials balance model (or any other reference technology). Furthermore, using the optimization results obtained under different flexibilities to adjust inputs and the amount of good outputs, we propose the following decomposition of the REM :

$$\frac{r^*_{En}}{r_{act}} = \frac{r^C_{Ex}}{r_{act}} \cdot \frac{r^*_{Ex}}{r^C_{Ex}} \cdot \frac{r^*_{En}}{r^*_{Ex}} \tag{2.15}$$

The first component ($r^C_{Ex}/r_{act}$) measures by how much the actual observed ratio can be increased relative to the best practice ratio if the inputs are fixed and the good output is not reduced. Hence, the measure captures ratio enhancements which relate to increases in the good and/or decreases in the bad output, potentially as a result of eliminating technical inefficiency. We refer to this component as *weak ratio efficiency* since a DMU may be capable of further increasing its ratio by further changing the good output and/or the inputs. The second component ($r^*_{Ex}/r^C_{Ex}$) measures the additional ratio improvements by a flexible choice of the produced amount of the good output implying that good outputs can be reduced below the exogenous constrained amount $y = y_i$. Therefore, this measure accounts for the possibility to increase the ratio by sacrificing the good output to further reduce the bad output. Since this component is similar to the allocative efficiency component in cost efficiency models (see Coelli et al. (2005)) we refer to it as *allocative ratio efficiency*. Finally, the third term ($r^*_{En}/r^*_{Ex}$) measures by how much the best practice ratio can be increased relative to $r^*_{Ex}$ when the DMU can freely choose the input mix. In this case, inputs can be increased or decreased compared to the actual amount of inputs used to further increase the optimal ratio of $y/b$.[7] Hence, we name this component *input ratio efficiency*.

For a graphical explanation of the REM we again consider the numerical example from table I. The observed ratio $r_{act}$ of DMU $C$ is defined by the slope of the dotted ray which intersects DMU $C$. The overall REM compares DMU $C$'s ratio to the maximal feasible ratio for the technology. Intuitively, the optimal ratio can be found by rotating the ray intersecting DMU $C$ as far "to the left" as possible in figure 2. This means that the optimal ray - both for the joint production model and the materials balance model - intersects DMU $A$ (i.e., DMU $A$ is overall REM efficient). However, to obtain the technologically optimal ratio DMU $C$ may possibly reduce its good output or alter its input consumption (which may lead to profit losses). To examine these aspects, we decompose the REM into its three components. The weak ratio efficiency component studies the difference between DMU $C$'s actual ratio and the optimal ratio for DMU $C$'s output set (i.e., for $x = 20$) when the good output (i.e., DMU $C$'s revenue) is not reduced. The joint production model suggests that DMU $C$ can obtain DMU $B$'s ratio without changing its input use or reducing the good output (recall that the JP model's output set for $x = 20$ is bounded by $0ABC80$). Hence, the model suggests that DMU $C$ is inefficient in terms of the first REM component. The materials balance model does, on the other hand, suggest that DMU $C$ cannot improve its ratio without reducing the good output; i.e., DMU $C$ is considered efficient in terms of the first REM component (recall that the MB model's output set for $x = 20$ is

[7] In line with conventional partial equilibrium analysis, we assume that the generating units do not compete for scarce resources. Hence, each generating unit determines the ratio-maximizing input mix independently of the other units.

bounded by $6A'C86$).

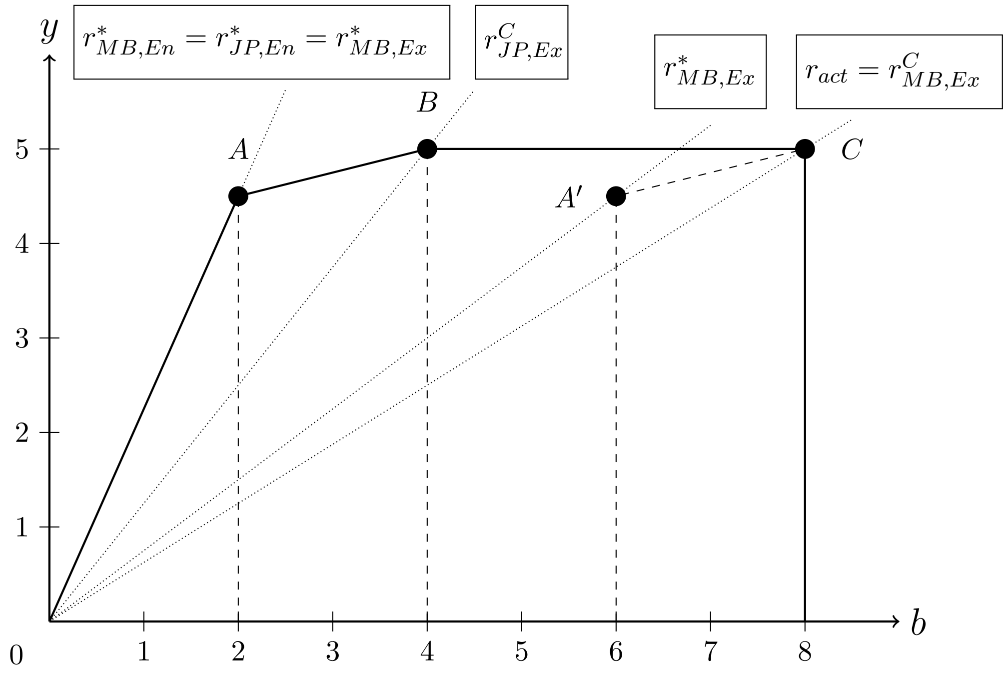


Figure 2: The REM and its decompositions

The allocative ratio efficiency component assesses whether further ratio improvements are feasible when the good output is reduced but the input consumption is unaltered. In this case, the joint production model suggests that DMU $C$'s ratio can be set equal to DMU $A$'s ratio, and thereby that the allocative ratio efficiency component amounts to the ratio of DMU $B$'s ratio to DMU $A$'s ratio. The materials balance model suggests that the optimal ratio equals DMU $A$'s "input adjusted" ratio (represented by the artificial datapoint $A'$ in figure 2) when inputs are fixed. Thus, the allocative ratio efficiency component amounts to the artificial DMU ($A'$)'s ratio to DMU $C$'s ratio.

The input ratio efficiency component considers whether DMU $C$'s ratio could be further improved by altering the input use. This is not the case for the joint production model. The reason for this result is that the free disposability assumption states that if DMU $A$'s ratio is feasible for $x = 10$, then DMU $A$'s ratio is also feasible for any larger input bundle. For the materials balance model, on the other hand, most of the potential for improving DMU $C$'s ratio comes from altering the input use. This is easily seen from figure 2 by comparing the ratio of DMU $A$ to DMU $A'$. Hence, our graphical example illustrates that there can be large differences in how the joint production model and the materials balance model describe the DMU's production possibilities.

## 2.4 Correcting the bias and regressing contextual variables

In the previous section we discussed how to use nonparametric methods to calculate the optimal ratios. Since the nonparametric estimation of the technology set is a subset of the true, but unknown production technology, the estimated optimal ratio is biased downwards (see Simar and Wilson (2008)). This result holds irrespective of whether the technology is constructed on the basis of the materials balance model or the joint production model. A bootstrapping approach to correct the bias for radial distance function estimations has been proposed by Simar and Wilson (1998) and for directional distance function estimations by Simar et al. (2012). Since our proposed ratio efficiency measure is not based on distance functions we cannot apply these approaches. Instead, we use subsampling methods to estimate and correct the bias in the optimal ratios. This approach has been proposed by Simar and Wilson (2011) who show that the subsampling approach (drawing $m < n$ observations without replacement) leads to consistent estimates of the bias given nonparametric frontier models.[8]

In the following we describe the algorithm to obtain the bias-corrected estimations of the optimal ratios. Note that we present the algorithm for the optimal ratio $y^*_{En}/b^*_{En}$, hence for the ratio to construct the overall REM. The ratios to construct the components of the REM can be bias-corrected following the same steps. In our presentation we modify the discussion of subsampling applied to distance functions in Simar and Wilson (2008, p. 451) to our ratio efficiency measure. The algorithm to obtain the bias-corrected ratios can be summarized as:

1. Use the original sample $\mathcal{X}_n = \{(\boldsymbol{x}_i, \boldsymbol{y}_i, \boldsymbol{b}_i), i = 1, \ldots, n\}$ to estimate the technology set $\widehat{T}$ based on the axioms of the joint production or the materials balance model. Use $\widehat{T}$ to estimate the optimal ratio $r^*_{En,i} = y^*_{En,i}/b^*_{En,i}$ for $i = 1, \ldots, n$ given the linearized programming problems defined above.

2. Draw without replacement $m < n$ observations from the original sample $\mathcal{X}_n$ and denote the resulting subsample $\tilde{\mathcal{X}}_m$.[9]

3. Use the subsample $\tilde{\mathcal{X}}_m$ to construct the technology and estimate the optimal ratio $\tilde{r}^*_{En,i} = \tilde{y}^*_{En,i}/\tilde{b}^*_{En,i}$ for each observation in the original sample $\mathcal{X}_n$.

4. Repeat steps 2 and 3 $B$ times and denote the results $\tilde{r}^*_{En,i,b}$ with $b = 1, \ldots, B$.

5. Use the subsampled ratio results to estimate the bias as

$$\widehat{bias}_B\left(r^*_{En,i}\right) = \left(\frac{m}{n}\right)^{\frac{2}{(m+k+s+1)}} \times \left[\frac{1}{B}\sum_{b=1}^{B} \tilde{r}^*_{En,i,b} - r^*_{En,i}\right] \tag{2.16}$$

   and estimate the bias-corrected optimal ratio as

$$r^*_{En,i,bc} = r^*_{En,i} - \widehat{bias}_B\left(r^*_{En,i}\right). \tag{2.17}$$

Since the bias correction introduces additional noise we follow Simar and Wilson (2008, p. 450) and correct for the bias only if $\frac{|\widehat{bias}_B(r^*_{En,i})|}{\widehat{\sigma}} > \frac{1}{\sqrt{3}}$, where $\widehat{\sigma}$ denotes the standard deviation of the optimal ratios based on the subsamples.

In the empirical part of our paper we are not only interested in estimating and decomposing the ratio efficiency measure but also in analyzing whether plant characteristics like age or size as well as

[8] Kneip et al. (2015) provide asymptotic results for the distribution of efficiency scores. However, their results are based on distance functions. Hence, in our analysis without distance functions and with a limited number of observations we rely on the more general subsampling technique.

[9] To obtain the optimal size of $m$ we follow the approaches by Politis et al. (2001) and Bickel and Sakov (2008). In these papers it is proposed to estimate the statistic of interest for each value in an interval around $m$ $(m-k, \ldots, m, \ldots, m+k)$ and to calculate a measure of variation for the results. This procedure is repeated for several values of $m$ and the value of $m$ with the minimal measure of variation is chosen for the subsampling. In our application we set $k = 2$ and estimate the median bias for each value in the interval. The variation is measured by the standard deviation of the results and we evaluate a grid of $(m = 30, 40, \ldots, 130)$. The optimal value for $m$ obtained by this method is $m = 100$.

other variables have a significant influence on the efficiency of the power plants.[10] Therefore, after estimating the efficiency we use regression methods to estimate the effects and test whether they are statistically significant. However, conventional inference based on the results of truncated regression with the efficiency measure as the dependent variable is not appropriate for this purpose (see Simar and Wilson (2013, pp. 304-320) for an overview of this issue and possible solutions). Simar and Wilson (2007) have shown that the correlation among the efficiency estimates which are based on nonparametric technology estimations leads to invalid inference results. To obtain valid estimates of the confidence intervals Simar and Wilson (2007) have proposed a double-bootstrap approach with the first bootstrap addressing the problem of bias-correcting the efficiency estimates and the second bootstrap providing valid statistical inference. For our regression explaining the results for the REM we combine the bias-correction based on subsampling as described above and the second bootstrap from the approach by Simar and Wilson (2007) to estimate the regression results. A detailed explanation on how to conduct the truncated second-stage regression and to bootstrap the results can be found in Simar and Wilson (2008, pp. 504-505). In our empirical application we use 2000 replications for each of the bootstraps discussed above.

Note that the validity of the regression approach by Simar and Wilson (2007) depends on a separability condition for the regressors and the technology set, implying that the regressors influence the efficiency results but not the shape of the technology.[11] To verify this condition, Daraio et al. (2010) have proposed a statistical test based on subsampling and a comparison of conditional and unconditional efficiency estimates using distance functions. In our model, which is not based on distance functions, this test cannot be readily applied.[12] We want to point out this caveat regarding the separability condition for our regression results.

# 3 Analysis of U.S. Power Plants

In this section we present the data and the results of the ratio efficiency analysis of U.S. power plants.

## 3.1 Constructing the dataset

We estimate the optimal ratios using a dataset containing 160 bituminous fired electricity generating units that were in operation in 2011.[13] Bituminous coal is an important energy source in the U.S., and accounted for about 43 percent of the electricity sector's total receipts of coal in 2011 (EIA (2013)). In

[10] See Simar and Wilson (forthcoming) for a survey of recent advances in statistical analyses of nonparametric frontier models.

[11] The estimation of partial frontier models (see Bădin et al. (2014) for an overview) is an alternative approach to account for the effects of plant characteristics. However, full frontier models as applied in this study may outperform partial frontier models (see Krüger (2012) for a comparison of the approaches based on Monte-Carlo simulations).

[12] We tried to test the condition based on distance functions, thus transforming our model. However, due to the inclusion of weakly disposable outputs, which are not accounted for in the original test, the conditional efficiency estimates are often infeasible, leading to no meaningful test results. Only in case of a CRS model and regression specification (1) defined in section 3.2 we were able to calculate the results. They indicate that the hypothesis of separability cannot be rejected for this specification.

[13] For more detailed information and definitions of coal-fired power plants and their generating units see Woodruff et al. (2012).

turn, coal fired electricity generation accounted for about 50 percent of the total domestic electricity generation.

Bituminous coal has very high sulfur content but similar carbon content to other types of coal.[14] Although there is currently no regulation for $CO_2$ emissions in place in the U.S. electricity sector, regulations for $SO_2$ and $NO_x$ emissions were implemented many years ago. The first air pollution control legislations were passed in the 1960s. Later, the Acid Rain Program (ARP) - a major program to control $SO_2$ and $NO_x$ from power plants - was implemented in 1995, and has been followed by other initiatives such as the Ozone Commission's cap-and-trade program for $NO_x$ and the Clean Air Interstate Rule (CAIR). Because bituminous coal firing is one of the largest sources of $SO_2$ and $NO_x$ emissions in the U.S. electricity sector, all the units in the dataset are regulated by the ARP. Most of the units are also regulated by the CAIR program.

We model technologies consisting of two inputs, bituminous coal and capital (proxied by generating capacity), which are used to produce electricity and $CO_2$.[15] Unlike other studies on polluting technologies (see e.g. Färe et al. (2005) and Murty et al. (2012)) we do not incorporate labor into our model. Since we are using generator-level data for our analysis we need precise data on the labor input. However, data on labor are only available on the plant-level (see Färe et al. (2005)) and hence we would need to rely on rough estimates of the labor input of the generating units. Moreover, recent studies (see Färe et al. (2013) and Hampf (2014)) have shown that data on labor input are very limited and hence we would be faced with a significant reduction in the number of observations in our sample if DMUs with missing labor data were to be excluded. However, a large number of generating units is important for the validity of our results since our analysis aims at providing information on the feasibility of the EPA standard. Moreover, Welch and Barnum (2009) argue that the labor input is proportional to the generating capacity. Therefore, by including capacity as an input we implicitly account for labor inputs as well. Hence, given these arguments we refrain from including labor explicitly in our analysis.

Information about the units' generating capacity is collected from the publicly available form EIA-860_Generator, while information on coal consumption, gross electricity generation, and $CO_2$ emissions is collected from the EPA database "Air Markets Program data". To ensure that we model a homogeneous technology we only include single-fueled units in the dataset; i.e. generating units that consume bituminous coal only. Second, we follow Mekaroonreung and Johnson (2012) and restrict our sample 1) to generating units with nameplate capacity larger or equal to 100 MW and 2) to pulverized coal-fired units. This results in a preliminary sample of 193 generating units.

There are three main categories of pulverized coal-fired units: subcritical, supercritical, and ultra-supercritical.[16] The differences between these categories relate to operating temperatures and pres-

[14] $NO_x$ formation is to a smaller degree dependent on the nitrogen content of the coal, but is primarily a function of temperature.

[15] One referee pointed out that the use of generating capacity as a proxy for capital ignores investments associated with abatement activities. We feel that this is appropriate in our analysis our since our efficiency analysis does not include pollutants for which abatement is a viable compliance strategy. Hence, including abatement inputs, but not accounting for pollution reduction, is likely to lead to biased efficiency measurement (see Färe et al. (2007b)). However, by correcting our efficiency results using second-stage regressions that account for regulated pollutants ($SO_2$ and $NO_x$) we correct for a potential bias due to pollution abatement. This procedure to explain efficiency is similar to the second-stage approach by Barros and Peypoch (2008).

[16] Pulverized boilers can also be separated into dry bottom and wet bottom units. Most of the DMUs in our dataset are

sures, which in turn have implications for operating efficiency. More specifically, the operating efficiencies of subcritical plants are usually less than the operating efficiencies of supercritical or ultra-supercritical plants. We omit all 32 supercritical units from the sample to avoid mistaking differences in the units' production technologies for potential for efficiency improvements. No units report that they are ultra-supercritical, but there are several missing values for pulverized coal-fired type in our dataset. Consequently, some of the units in the sample may be supercritical or ultra-supercritical. We use a battery of non-parametric tests (the Kolomogorov-Smirnov test, ANOVA, the Wilcoxon rank-sum test, and the median test, hereafter referred to as "the non-parametric tests") to consider whether the ratios of electricity to $CO_2$ emissions differ for the reported subcritical units and the units that do not report their type. All tests indicate that there are no statistical differences between the two groups' $CO_2$ efficiencies, and we do therefore not exclude the generating units that do not report their type from the sample. This results in a dataset containing 161 electricity generating units in operation in 2011.

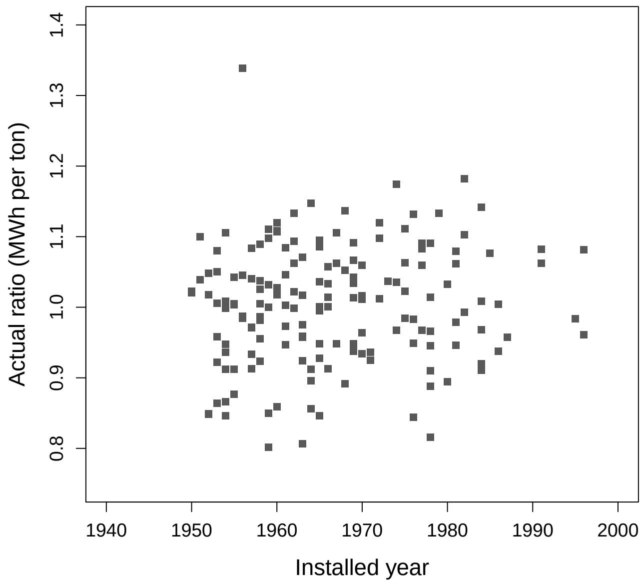


Figure 3: Actual ratios - MWh produced per ton of $CO_2$ emitted

Next, we inspect the actual ratios of electricity to carbon dioxide emissions to control for potential outliers in our dataset. As can be seen from figure 3, one unit by far outperforms the other units in terms of its electricity to $CO_2$ ratio. This particular unit's ratio is 33 percent higher than the average ratio and 13 percent higher than the second most efficient unit's ratio. We estimate the optimal ratios with and without the identified outlier, and use the non-parametric tests to consider whether including the unit in the dataset influences the results. The tests strongly support that including the outlier influences the results and we therefore omit it from the dataset.[17] This leads to a final sample size of

dry bottom units. We use the nonparametric tests to consider whether the DMUs' observed electricity to $CO_2$ ratios and the empirical results differ for dry and wet bottom units, but we are unable to detect any differences. Therefore, we do not exclude wet bottom units from the dataset.

[17] We checked whether this observation is only an outlier for the specific year 2011. However, we also found for the years 2010 ($y/b = 1.25$ MWh per ton) and 2012 ($y/b = 1.28$ MWh per ton) that this observations is an outlier which always exhibits the largest ratio of $y/b$.

160 analyzed generating units.

By undertaking the steps above to ensure that the dataset contains homogeneous DMUs we believe that our study offers substantial advancement relative to other comparable studies on polluting technologies. The common practice is to merge units with different production technologies and/or units that consume different types of fossil fuels into one dataset.[18] For example, the selection criterion used for a popular dataset on coal fired power plants (see e.g. Färe et al. (2007b)) is that at least 95 percent of the plants' energy inputs must come from coal. The power plant's technology type or the qualities of their fuels are not emphasized, and the units are allowed to consume oil and natural gas in addition to coal. We therefore question whether efficiency analyses based on this and similar datasets reflect actual possibilities for efficiency improvements or whether they reflect technological differences among the units (see Heshmati et al. (2012) for a further analysis of the importance of accounting for heterogeneity in the technologies when analyzing power plant efficiency).

In order to undertake the regression analysis we add a variable containing the generating units startup year to the dataset. This variable is collected from the form EIA-860_Generator. Second, emissions of $SO_2$ and $NO_x$ are collected from the "Air Markets Program" database. Finally, $CO_2$ emission factors are calculated by dividing the generating units' $CO_2$ emissions on their bituminous coal consumption. This approach is in line with the materials balance principle from equation (2.2) since there is no end-of-pipe abatement taking place for $CO_2$. Summary statistics of the dataset are reported in table II.

Table II: Summary statistics (160 DMUs)

| Variable | Units | Mean | St.dev | Min | Max |
|---|---|---|---|---|---|
| Fuel | mmBTUs | 16 100 000.00 | 14 700 000.00 | 57 417.69 | 4 900 000.00 |
| Capacity | MW | 337.92 | 231.84 | 100.00 | 1 425.60 |
| Electricity | MWh | 1 696 715.00 | 1 616 519.00 | 5 884.10 | 8 541 296.00 |
| $CO_2$ | Tons | 1653 226.00 | 1 509 647.00 | 5 890.97 | 7 686 116.00 |
| $SO_2$ | Tons | 4 153.28 | 6 513.48 | 34.23 | 57 308.22 |
| $NO_x$ | Tons | 1 625.39 | 1 363.38 | 15.73 | 8 438.45 |
| Installed year | Year | 1 966.73 | 10.83 | 1 950.00 | 1 996.00 |
| Emission factor | Ton/mmBTU | 0.10 | 0.00 | 0.10 | 0.11 |

## 3.2 Results of the efficiency analysis

Having presented the dataset we now turn to the empirical results. First, we present the estimated maximal feasible MWhs to $CO_2$ ratios in figure 4. The overall figure is made up of 4 sub-figures, each containing histograms for 1) the optimal ratios in the scenario where the good output cannot be reduced (i.e, be reduced below the actual produced amount $y_i$ in order to reduce emissions) to reduce $CO_2$ emissions (the weak efficient ratios), 2) for the scenario where the good output may be reduced to reduce $CO_2$ emissions (the allocative efficient ratios), and 3), the scenario where the DMUs may

[18] One exception is the recent study by Mekaroonreung and Johnson (2012), which also utilizes a dataset for bituminous generating units. To our understanding, Mekaroonreung and Johnson did not distinguish between different pulverized coal-fired technologies (i.e. subcritical, supercritical, and ultra-supercritical units) when compiling their dataset.

alter their input mix to improve the optimal ratio (the input efficient ratios). The 4 sub-figures report the results for the two model specifications (JP and MB) and the two scale assumptions (VRS and CRS).[19]

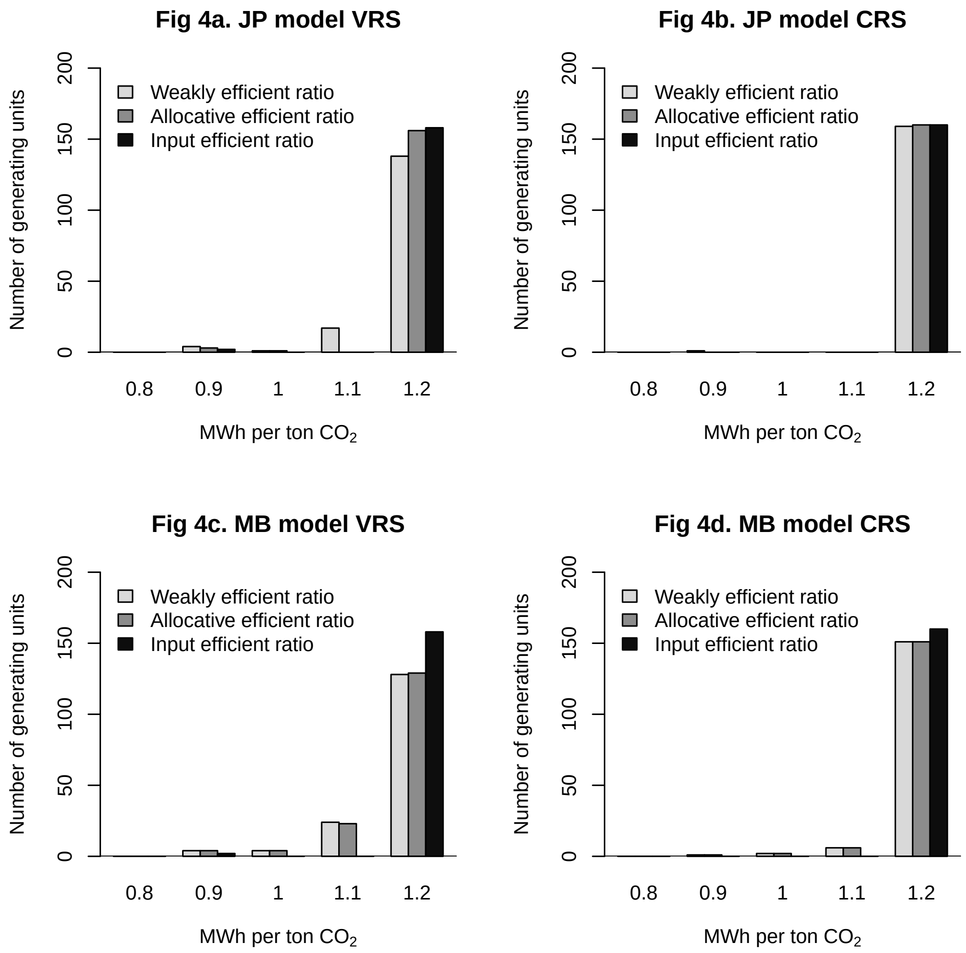


Figure 4: Optimal ratios

The first noticeable aspect of figure 4 is that all four model specifications suggest a maximal feasible ratio of approximately 1.18, which corresponds to the maximal ratio in the dataset. This optimal ratio should be compared to EPA's proposed $CO_2$ emission standard of 1000 pounds of $CO_2$ per megawatt-hour for new plants or, stated differently, 0.5 tons of $CO_2$ per megawatt-hour produced. Since we consider its inverse, the generating units are required to have a ratio of $1/0.5 = 2$ MWh per ton $CO_2$ or higher to comply with the EPA's standard. Clearly, the optimal ratio of 1.18 falls short of this standard (it is 40% below the EPA standard), thereby indicating that introducing the proposed standard will have significant economic implications for the existing electricity producers by forcing them to retire or to invest in end-of-pipe technologies (Carbon Capture and Storage) that are still in their infancies.

According to figure 4, most of the DMUs in the dataset would be capable of achieving the best practice ratio of 1.18. This result is consistent across both the production models (JP and MB) and the scale

[19] Detailed results for each generating unit can be obtained from the authors upon request.

assumptions (VRS and CRS). However, some differences in the results across the 4 different model specifications should be pointed out.

First, the DMUs' maximal feasible ratios are estimated to be slightly higher for the CRS specifications than for the corresponding VRS specifications. For the CRS specifications most generating units have an optimal ratio of 1.18, also in the cases where the optimal ratios are calculated without altering the DMUs' input mixes. The reason for this is that the CRS assumption implies that if a certain optimal ratio is feasible for a given input vector, then that optimal ratio is also feasible for any larger input vector (and their corresponding output sets). In other words, the (globally) optimal ratio of 1.18 appears to be feasible for most of the evaluated output sets under CRS.

Second, the maximal ratios calculated by the MB model are found to be slightly less than the corresponding ratios calculated by the JP model (irrespective of the scale assumption), except for the model specifications where the inputs are allowed to be reallocated to improve the optimal ratio. This result can be attributed to the difference in the models' assumptions about input disposability. We refer to figures 1 and 2 for details. We also note that the small differences in the results for the JP and MB models may be attributed to our choice of efficiency measure, namely the maximal ratios. Other measures - such as directional distance functions (see Färe and Grosskopf (2004) for a theoretical discussion and Weber and Domazlicky (2001) for an application) - may result in larger differences among the JP and MB models' result, in particular because the slack variables for electricity and $CO_2$ may not be zero in the solution to the programming problems for the distance functions, unlike in the programming problem for the maximal ratio for the JP model.

Third, the two models differ slightly in terms of how they capture the (potential) economic trade-offs related to reducing $CO_2$ emissions. From figure 4 we see that the JP and MB models both appear to suggest that a positive trade-off between electricity generation and $CO_2$ emissions exists, since the light grey bars are not parallel to the medium grey bars. As mentioned in section 2, such a trade-off is likely to occur when emission reductions solely take place by diverting resources from intended production to pollution control, in particular to end-of-pipe abatement activities. Such controls for $CO_2$ are not adopted by the U.S. bituminous producers, and the trade-off should therefore be close to zero. Note that the trade-off suggested by the MB model appears to be smaller than the corresponding trade-off suggested by the JP model. We use the non-parametric tests to examine statistical differences in the optimal ratios calculated with and without the possibility of reducing the good output to carefully examine how the two production models portray the proposed economic trade-off. All four tests are unable to reject the null hypothesis of no differences among the ratios for the MB model, both under VRS and CRS. For the JP model, on the other hand, all the tests strongly reject the null-hypothesis for the VRS specification and two tests (the Kolomogorov-Smirnov and the Wilcoxon rank-sum tests) strongly reject the null hypothesis under CRS. Consequently, the MB model seems to be more appropriate to case studies where end-of-pipe abatement is not common.

So far we have discussed which optimal ratios could be achieved if the generating units adopt best practices. The next step is to compare current practices with best practices. Hence, figure 5 provides cumulative plots of the REM and its decompositions. As before, 1) the light grey bars indicate the possible ratio improvements for given inputs and without the possibility to reduce the good output, 2) the medium grey bars indicate additional ratio improvements by reducing the good output (for fixed inputs), while 3) the dark grey bars indicate additional ratio improvements by reallocating inputs. On the horizontal axes the generating units are listed in order of the least to the most inefficient unit. The vertical axes indicate the magnitudes of the REM, i.e. they report the percentage possible increases in the DMUs' actual ratios.

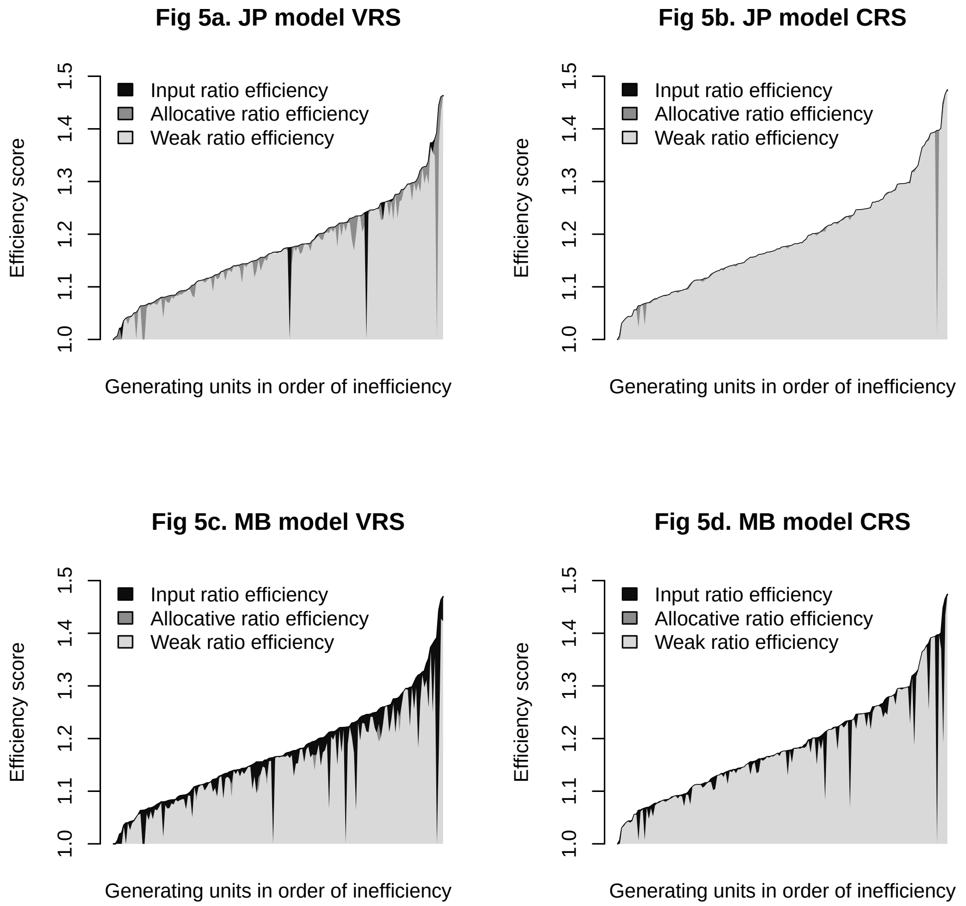


Figure 5: Results for the ratio efficiency measure

Since the optimal ratios in the scenario where inputs can be allocated freely are similar for the JP and MB models it follows readily that the overall REM is similar for the two reference technologies.[20] Our DEA results indicate that the generating unit's ratios of MWh produced to tons of $CO_2$ emitted could, on average, be improved by 18 percent according to the VRS model specifications and 19 percent according to the CRS model specifications - if the generating units adopt best practices. Stated differently, the average actual practice ratio of electricity to $CO_2$ is approximately $100 \cdot (1 - 1/1.185) = 15.3\%$ below the best-practice ratio. The results also indicate that the "worst practice" generating unit could be able to improve its ratio by 46 percent according to the VRS specifications and 47 percent according to the CRS specifications.

From figure 5 it is clear that the light grey bars are dominating, i.e. most of the potential for improving

[20] Note that given our analyzed empirical specification both models (JP and MB) lead to the same results of the overall REM under CRS. A proof of this equivalence can be found in appendix A.

the generating units electricity to $CO_2$ ratios comes from improvements in the weak ratio efficiency. While it is the dominating source of MWhs to $CO_2$ ratio improvements, the remaining potential for improvements comes either from reducing the good output or from altering the current input consumption. It is evident from figure 5 that the occurrences of dark grey bars are far more frequent for the MB model's estimates than for the JP model's estimates, while the occurrences of medium grey bars are far more frequent for the JP model's estimates than for the MB model's estimates. Simply said, the MB model does not indicate a positive trade-off among electricity and $CO_2$ (implying that additional ratio improvements must therefore come from input reallocation), while the JP model does. On the basis of this finding we again conclude that the MB model paints a more accurate picture of the possible sources of ratio improvements for the case study at hand.

To indicate the environmental gains from adopting best practices we undertake a simple calculation. We multiply the inverse of the estimated optimal ratios (calculated for fixed inputs and without possibilities to reduce revenues) with the actual MWhs produced by the generating units to obtain estimates of $CO_2$ emissions in the case where all units operate efficiently.[21] Table III presents the sums of the estimated emissions and the corresponding total and percentage reductions in $CO_2$ emissions relative to the units' actual emissions, which for the 160 generating units amount to 264.52 million tons of $CO_2$. For comparison, we also include the corresponding estimates if all units complied with EPA's proposed standard of 0.5 tons of $CO_2$ per megawatt-hour produced.

Table III: $CO_2$ emissions and savings

| | JP VRS | JP CRS | MB VRS | MB CRS | EPA standard |
|---|---|---|---|---|---|
| Aggregate $CO_2$ emissions (mill. Tons) | 232.34 | 230.77 | 234.75 | 233.09 | 135.74 |
| Aggregate $CO_2$ reductions (mill. Tons) | 32.18 | 33.75 | 29.77 | 31.43 | 128.78 |
| Percentage $CO_2$ reductions | 12.16 | 12.76 | 11.25 | 11.88 | 48.68 |

The JP model reports a greater potential for efficiency improvements than the MB model, in particular because the JP model puts less emphasis on input reallocations to reduce emissions than the MB model. However, both models suggest that the gains from efficiency improvements on $CO_2$ emissions could be substantial, averaging at about 12 percent reductions in current emissions. Yet, these savings are far from the corresponding emission reductions that would occur if the generating units were able to comply with the proposed EPA standard.

While the potential for reducing $CO_2$ emissions by efficiency improvements at first sight appears to be vast, the differences between the DMUs' performances may not only be related to differences in management practices. There is also a possibility that contextual factors - factors which are outside of the control of the generating units - play an important role in determining the spread in the efficiency scores. Identifying these factors are important, both for achieving a better understanding of our empirical results and for establishing factors that should be taken into account when designing new

[21] Given that the largest share of the potential for ratio improvements comes from weak ratio efficiency improvements this procedure allows us to quantify nearly all reductions potentials while accounting for the constraints on the inputs and the good output.

regulations for $CO_2$ emissions. We undertake a second-stage regression analysis to shed some light on this matter, emphasizing the role which the generating units' age, sizes (generating capacities), and existing regulations for $SO_2$ and $NO_x$ play for the efficiency results. In the regressions we analyze the effects of these contextual factors on the weak ratio efficiency. This allows us to quantify feasible emission standards based on improvements in managerial inefficiencies, and further to compare the resulting $CO_2$ emissions to the best practice $CO_2$ emissions in table III which do not account for the influence of contextual variables on the possibilities for efficiency improvements.

In table IV the regression results for the 4 analyzed models (the JP and the MB model under CRS and VRS) are presented. We consider two different specifications for the regression model. In the first model we solely address the effects of the age (installed year) and size (capacity) of the generating units. In the second model we include interaction terms of the age and the ratio of electricity to $SO_2$ as well as the ratio of electricity to $NO_x$ in the regression. This is done to analyze whether the increased stringency of environmental regulation for local air pollutants (proxied by their emission ratios) over time has influenced the $CO_2$-efficiencies of the generating units.[22]

End of pipe abatement for $SO_2$ - also known as scrubbers - has become one of the most common compliance strategies for U.S. power plants (see Ellerman (2003) or Rødseth and Romstad (2014)). The use of scrubbers to reduce $SO_2$ can affect the ratio of electricity to $CO_2$ in two different ways. First, scrubbers consume a non-negliable amount of electricity during operation. Second, chemical processes to reduce $SO_2$ can lead to additional $CO_2$ (see Agee et al. (2014)). In our regression we can only account for the latter effect since we are using gross electricity as the good output when estimating the REM. We do so because the evaluated EPA regulations are also based on gross electricity production.

While compliance with $SO_2$ regulations largely has been achieved by fuel switching and pollution control, the introduction of the Acid Rain Program in 1995 also led most power producers under environmental legislation to install low $NO_x$-burners to reduce nitrogen oxides (see Swift (2001)). Low $NO_x$-burners reduce the peak flame temperature and thereby also reduce the formation of $NO_x$.

Table IV presents the estimated coefficients and their standard errors. We analyzed several functional specifications and found that a log-log specification yields the largest explanatory power (as measured by the $R^2$ values). Hence, the coefficients presented in table IV are elasticities. Note that a negative coefficient is associated with an increase in efficiency (the weak ratio efficiency becomes closer to one).

Based on regression model 1 we find that larger generating units are more efficient than smaller units. Hence, significant economies of scale with regard to the ratio of electricity to $CO_2$ exist, which suggests that larger generating units would be better equipped to meet new $CO_2$ regulations.[23] Rather surprisingly, we find that this effect is significant for the analysis under CRS as well as VRS. Since the analysis under VRS only accounts for technical efficiency and does not account for scale efficiency we would expect the coefficient not to be statistically different from zero under VRS. Second, we find that newer generating units are in general more inefficient than older units. At first, this may appear to be a counterintuitive result. However, we expect this result to be related to the stringency of the

[22] Note that we have also conducted the regressions in model 2 by including the $SO_2$ and the $NO_x$ ratio as additional independent variables. However, due to multicollinearity the standard errors inflated and the regressions did not lead to any statistically meaningful results.

[23] It has long been debated that environmental regulations hamper competition by increasing the optimal plant size (see e.g. Pashigan (1984)). Our finding bares resemblance to this result.

environmental regulations for $NO_x$ and $SO_2$ which has increased over time.

To evaluate whether this is indeed the case, regression model 2 includes the interaction terms for the age of the units and the ratios of electricity to $SO_2$ and $NO_x$. The results for regression model 2 show that when accounting for these interactions the surprising result from regression model 1 with respect to the negative effect of size on the units' performances under VRS is not statistically significant anymore. Moreover, the interaction term between the age and the $NO_x$ ratio has a significant negative coefficient, thereby indicating that an increase in the stringency of regulation reduces the effect of age on efficiency. Thus, $CO_2$ and $NO_x$ appear to be complements, which means that an increase in the regulatory stringency for $NO_x$ leads to further improvements in $CO_2$ emissions. Therefore, our results are in line with the findings by Holland (2010) who shows in an analysis of Californian power plants that $NO_x$ and $CO_2$ are complements.[24] We do not find a corresponding significant relationship for the units' age and $SO_2$ stringency. This may result because we consider gross electricity and therefore do not fully capture the influences of $SO_2$ reductions on $CO_2$ efficiencies.

These results provide an important lesson, namely that $CO_2$ and $NO_x$ are technologically related and hence that regulations which are implemented for one of the pollutants have impacts for the other pollutant. This suggests that socially optimal allocations for the pollutants are unlikely to be achieved by implementing regulations on a pollutant-by-pollutant basis.

Furthermore, the regression results provide valuable information on which emission standard could be achieved by reducing managerial inefficiencies. To estimate a technically feasible emission standard for the average generating unit we utilize the regression results to calculate the managerial inefficiency for the average generating unit and multiply this efficiency measure with the actual ratio of electricity to $CO_2$ of the generating unit. We further multiply the feasible emission standards with the total MWhs generated by the 160 units to obtain a tentative measure of the $CO_2$ emissions under managerial efficiency. The results for the different models are presented in table V.

The results show that the feasible emission standard based on improving the managerial efficiency of the generating unit (approx. 1943 pounds of $CO_2$ per MWh of electricity) is far larger than emission standard of the EPA (1000 pounds $CO_2$ per MWh) irrespective of which model is used to evaluate the efficiency of the units. Moreover, comparing the results to the average actual ratio (1997 pounds of $CO_2$ per MWh of electricity) shows that the efficiency improvement possibilities which are due to managerial deficits in the plant are very small. We find that the $CO_2$ emissions under managerial efficiency are approximately 0.3 percent lower than the actual emissions, whereas the DEA results in table III suggested that a 12 percent reduction in the $CO_2$ emissions would be feasible. Hence, regulatory actions which aim at improving the efficiency of electricity plants by imposing emission standards can only exploit a very small amount of inefficiencies without forcing the plants to shut down operations or to invest in completely new generating capacities.

[24] An alternative approach to examine the substitutability among pollutants, using the directional distance function to calculate the Morishima elasticity of transformation, has been proposed by Färe et al. (2012). They analyze the power production in the US, but emphasize the substitutability among $SO_2$ and $NO_x$.

Table IV: Regression results

| | CRS | | | | VRS | | | |
|---|---|---|---|---|---|---|---|---|
| | Joint production | | Materials balance | | Joint production | | Materials balance | |
| | Model 1 | Model 2 | Model 1 | Model 2 | Model 1 | Model 2 | Model 1 | Model 2 |
| Constant | −42.470*** | −33.520*** | −35.420*** | −26.890** | −42.288*** | −37.616** | −37.184** | −32.641** |
| | (13.209) | (13.473) | (13.420) | (13.411) | (14.993) | (15.421) | (14.955) | (15.361) |
| Capacity | −0.096*** | −0.071*** | −0.090*** | −0.065*** | −0.057*** | −0.032 | −0.051*** | −0.026 |
| | (0.015) | (0.018) | (0.016) | (0.018) | (0.017) | (0.021) | (0.017) | (0.021) |
| Installed year | 5.692*** | 4.519** | 4.756*** | 3.640** | 5.636*** | 5.028** | 4.956** | 4.366** |
| | (1.750) | (1.784) | (1.742) | (1.776) | (1.986) | (2.042) | (1.981) | (2.034) |
| Installed year* | | 0.001 | | 0.001 | | 0.001 | | 0.001 |
| $SO_2$ ratio | | (0.008) | | (0.008) | | (0.001) | | (0.001) |
| Installed year* | | −0.005*** | | −0.005*** | | −0.004* | | −0.004* |
| $NO_X$ ratio | | (0.002) | | (0.002) | | (0.002) | | (0.002) |
| $R^2$ | 0.215 | 0.250 | 0.202 | 0.237 | 0.067 | 0.091 | 0.054 | 0.078 |
| Adj. $R^2$ | 0.210 | 0.236 | 0.197 | 0.223 | 0.061 | 0.073 | 0.048 | 0.060 |

Note: Significance levels are obtained from confidence intervals based on Simar and Wilson (2007) using 2000 bootstrap replications.
Standard errors are presented in parentheses.
*** indicates significance at the 1% level.
** indicates significance at the 5% level.
* indicates significance at the 10% level.

Table V: Feasible emission standards

| Technology model | Feasible emission standard (pounds of $CO_2$ per MWh electricity) | Aggregate $CO_2$ emissions (mio. tons of $CO_2$) |
|---|---|---|
| Joint production (CRS) | 1943 | 263.80 |
| Materials balance (CRS) | 1944 | 263.84 |
| Joint production (VRS) | 1943 | 263.68 |
| Materials balance (VRS) | 1942 | 263.60 |

## 4 Conclusion

In this paper we have conducted an efficiency analysis of coal-fired U.S. power plants. Using a set of 160 homogeneous electricity generating units we addressed whether the proposed EPA regulatory standard of 1000 pounds of carbon dioxide emissions per megawatt hour would be feasible for existing generating units. To analyze the feasibility of this standard we have constructed a new efficiency measure which evaluates the optimal ratio of good to bad outputs. Moreover, it allows disentangling efficiency improvements based on different degrees of flexibility in the choice of good outputs and inputs. We estimated the efficiency measure and compared the results using two different technology models: the joint production model based on weak disposability of pollutants by Färe et al. (1989) and the recently developed materials balance model by Rødseth (2014a) which is based on the assumption of weak G-disposability.

Our results show that even if all generating units were able to adopt best practices their ratios of electricity to carbon dioxide would still be 40% below the EPA standard. Moreover, even the adoption of best practices seems beyond the capability of most plants since our regression results show that the efficiency results are significantly influenced by contextual variables like the age of the plants. Hence, a large share of efficiency improvements is only achievable in the long run and would be associated with significant costs related to the restructuring of the power plants.

Therefore, our results strengthen the findings by Kotchen and Mansur (2014) who are very pessimistic about the possibility to achieve the proposed emission standard given the actual ratios of the power plants. Our results add to the results of Kotchen and Mansur (2014) by showing that even the adoption of best practices would not allow the power plants to meet the emission standards. Moreover, we were able to show that significant interdependences between $CO_2$ and $NO_x$ emissions exist. Hence, a new regulation for $CO_2$ is likely to affect other regulated pollutants, suggesting that regulations set on a pollutant-by-pollutant basis are unlikely to result in socially optimal allocations.

In the light of these results we strongly question imposing the proposed EPA standard on existing power plants. However, this does not lead to the conclusion that no standard at all should be implemented. Our findings show that efficiency enhancement potentials exist which are not due to contextual variables and thus are under the control of the plant operators. However, these potentials are rather small and our efficiency and regression results show that a feasible emission standard for the average existing generating unit should not be lower than 1943 pounds of $CO_2$ per megawatt hour of electricity. Future research should therefore address how the costs of restructuring the coal-fired power plants compare to the damage costs of carbon dioxide emissions. Second, the contributions of more flexible regulatory schemes - such as emissions averaging and tradable quotas - in reducing the power producers' compliance costs should also be taken into account.

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# Appendix A

**Linearization of the optimization problems (scalar cases)**

*Joint production model under CRS (2.5)*

The optimization model is given by

$$\begin{aligned}
\max_{y,b,\boldsymbol{\lambda}} \quad & \frac{y}{b} \\
\text{s.t.} \quad x_i^P &\geqq \boldsymbol{x}^{P^T}\boldsymbol{\lambda} \\
x_i^{NP} &\geqq \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} \\
y &\leqq \boldsymbol{y}^T\boldsymbol{\lambda} \\
b &= \boldsymbol{b}^T\boldsymbol{\lambda} \\
y, b &\geqq 0 \\
\boldsymbol{\lambda} &\geqq \boldsymbol{0}.
\end{aligned} \tag{2.5}$$

In the optimum the constraints on the good and the bad output hold with equality since $y$ and $b$ can be freely chosen. Replacing $y$ and $b$ in the objective function by these equalities leads to the transformed programming problem

$$\begin{aligned}
\max_{\boldsymbol{\lambda}} \quad & \frac{\boldsymbol{y}^T\boldsymbol{\lambda}}{\boldsymbol{b}^T\boldsymbol{\lambda}} \\
\text{s.t.} \quad x_i^P &\geqq \boldsymbol{x}^{P^T}\boldsymbol{\lambda} \\
x_i^{NP} &\geqq \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} \\
\boldsymbol{\lambda} &\geqq \boldsymbol{0}.
\end{aligned} \tag{A.1}$$

Denoting $\boldsymbol{z} = \frac{1}{\boldsymbol{b}^T\boldsymbol{\lambda}}\boldsymbol{\lambda}$ and $w = \frac{1}{\boldsymbol{b}^T\boldsymbol{\lambda}}$ as well as applying the transformation by Charnes and Cooper (1962) leads to (2.6).

*Joint production model under VRS (2.7)*

Model (2.7) can be reformulated as

$$\begin{aligned}
\max_{y,b,\boldsymbol{\lambda},\theta} \quad & \frac{y}{b} \\
\text{s.t.} \quad x_i^P &\geqq \boldsymbol{x}^{P^T}\boldsymbol{\lambda} \\
x_i^{NP} &\geqq \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} \\
\tfrac{1}{\theta}y &\leqq \boldsymbol{y}^T\boldsymbol{\lambda} \\
\tfrac{1}{\theta}b &= \boldsymbol{b}^T\boldsymbol{\lambda} \\
\boldsymbol{1}^T\boldsymbol{\lambda} &= 1 \\
\tfrac{1}{\theta} &\geqq 1 \\
y, b &\geqq 0 \\
\boldsymbol{\lambda} &\geqq \boldsymbol{0}.
\end{aligned} \tag{A.2}$$

Multiplying each constraint by $\theta$, denoting $\boldsymbol{q} = \boldsymbol{\lambda}\theta$ and inserting the constraints on $y$ and $b$ (which again hold with equality) into the objective function leads to

$$\begin{aligned} \max_{\boldsymbol{q},\theta} \quad & \frac{\boldsymbol{y}^T\boldsymbol{q}}{\boldsymbol{b}^T\boldsymbol{q}} \\ \text{s.t.} \quad \theta x_i^P &\geqq \boldsymbol{x}^{P^T}\boldsymbol{q} \\ \theta x_i^{NP} &\geqq \boldsymbol{x}^{NP^T}\boldsymbol{q} \\ \boldsymbol{1}^T\boldsymbol{q} &= \theta \\ 1 &\geqq \theta \\ \boldsymbol{q} &\geqq \boldsymbol{0}. \end{aligned} \tag{A.3}$$

Inserting $\boldsymbol{1}^T\boldsymbol{q} = \theta$ in the constraints for the inputs and the constraint on $\theta$ the programming problem can be reformulated as

$$\begin{aligned} \max_{\boldsymbol{q}} \quad & \frac{\boldsymbol{y}^T\boldsymbol{q}}{\boldsymbol{b}^T\boldsymbol{q}} \\ \text{s.t.} \quad \left(\boldsymbol{x}^P - x_i^P\right)^T\boldsymbol{q} &\leqq 0 \\ \left(\boldsymbol{x}^{NP} - x_i^{NP}\right)^T\boldsymbol{q} &\leqq 0 \\ \boldsymbol{1}^T\boldsymbol{q} &\leqq 1 \\ \boldsymbol{q} &\geqq \boldsymbol{0}. \end{aligned} \tag{A.4}$$

Denoting $\boldsymbol{g} = \frac{1}{\boldsymbol{b}^T\boldsymbol{q}}\boldsymbol{q}$ and $h = \frac{1}{\boldsymbol{b}^T\boldsymbol{q}}$ and applying the Charnes-Cooper transformation leads to programming problem (2.8).

*Materials balance model under CRS (2.9)*

The non-linear programming problem for the MB model under CRS is given by

$$\begin{aligned} \max_{y,b,\epsilon_{x^P},\epsilon_{x^{NP}},\epsilon_y,\epsilon_b,\boldsymbol{\lambda}} \quad & \frac{y}{b} \\ \text{s.t. } x_i^P &= \boldsymbol{x}^{P^T}\boldsymbol{\lambda} + \epsilon_{x^P} \\ x_i^{NP} &= \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} + \epsilon_{x^{NP}} \\ y &= \boldsymbol{y}^T\boldsymbol{\lambda} - \epsilon_y \\ b &= \boldsymbol{b}^T\boldsymbol{\lambda} + \epsilon_b \\ \epsilon_b &= s_{x^P}\epsilon_x^P + s_y\epsilon_y \\ y, b &\geqq 0 \\ \epsilon_{x^P}, \epsilon_{x^{NP}}, \epsilon_y, \epsilon_b &\geqq 0 \\ \boldsymbol{\lambda} &\geqq \boldsymbol{0}. \end{aligned} \tag{2.9}$$

Since the good output in our empirical application (electricity) does not contain any pollution ($s_y = 0$) and the non-polluting input $x^{NP}$ does not contain any pollution by definition, we can remove the slacks $\epsilon_y$ and $\epsilon_{x^{NP}}$ and replace the corresponding equations by inequalities. Replacing the constraints on $y$

and $b$ in the objective function leads to

$$\begin{aligned}
\max_{\epsilon_{x^P}, \epsilon_b, \boldsymbol{\lambda}} \quad & \frac{\boldsymbol{y}^T \boldsymbol{\lambda}}{\boldsymbol{b}^T \boldsymbol{\lambda} + \epsilon_b} \\
\text{s.t. } x_i^P &= \boldsymbol{x}^{P^T} \boldsymbol{\lambda} + \epsilon_{x^P} \\
x_i^{NP} &\geqq \boldsymbol{x}^{NP^T} \boldsymbol{\lambda} \\
\epsilon_b &= s_{x^P} \epsilon_x^P \\
\epsilon_{x^P}, \epsilon_b &\geqq 0 \\
\boldsymbol{\lambda} &\geqq \mathbf{0}.
\end{aligned} \tag{A.5}$$

Rearranging the constraint on the polluting input and combining it with the materials balance restriction leads to $\epsilon_b = s_{x^P} \left( x_i^P - \boldsymbol{x}^{P^T} \boldsymbol{\lambda} \right)$. Inserting this expression in the objective function and rearranging leads to

$$\begin{aligned}
\max_{\boldsymbol{\lambda}} \quad & \frac{\boldsymbol{y}^T \boldsymbol{\lambda}}{\left( \boldsymbol{b} - s_{x^P} \boldsymbol{x}^P \right)^T \boldsymbol{\lambda} + s_{x^P} x_i^P} \\
\text{s.t. } x_i^{NP} &\geqq \boldsymbol{x}^{NP^T} \boldsymbol{\lambda} \\
\boldsymbol{\lambda} &\geqq \mathbf{0}.
\end{aligned} \tag{A.6}$$

Denoting $\boldsymbol{c} = \frac{1}{\left( \boldsymbol{b} - s_{x^P} \boldsymbol{x}^P \right)^T \boldsymbol{\lambda} + s_{x^P} x_i^P} \boldsymbol{\lambda}$ and $v = \frac{1}{\left( \boldsymbol{b} - s_{x^P} \boldsymbol{x}^P \right)^T \boldsymbol{\lambda} + s_{x^P} x_i^P}$ and applying the Charnes-Cooper transformation leads to the programming problem (2.10).

*Linearization of (2.11)*

The derivation of the linearization of (2.11) is equal to the linearization of (2.9) with the restriction $\mathbf{1}^T \boldsymbol{\lambda} = 1$ added to the programming problem. Hence, it is not repeated here.

**Equivalence of the overall REM for the JP and the MB model under CRS (scalar case)**

In the following we demonstrate that given the model applied in our empirical analysis the REM leads to the same results for the joint production and the materials balance approach. The equivalence holds for the overall efficiency under constant returns to scale. However, the results for the decomposition as well as the analysis under variable returns to scale may differ.

In our empirical specification we include a single polluting input, a single non-polluting input as well as a single good and a single bad output. Hence, the joint production model under CRS reads as

$$\begin{aligned}
\max_{y, b, x^P, \boldsymbol{\lambda}} \quad & \tfrac{y}{b} \\
\text{s.t.} \quad x^P &\geqq \boldsymbol{x}^{P^T} \boldsymbol{\lambda} \\
x_i^{NP} &\geqq \boldsymbol{x}^{NP^T} \boldsymbol{\lambda} \\
y &\leqq \boldsymbol{y}^T \boldsymbol{\lambda} \\
b &= \boldsymbol{b}^T \boldsymbol{\lambda} \\
y, b, x^P &\geqq 0 \\
\boldsymbol{\lambda} &\geqq \mathbf{0}.
\end{aligned} \tag{A.7}$$

Since $x^P \geqq \boldsymbol{x}^{P^T} \boldsymbol{\lambda}$ does not constrain the optimal results due to the free choice of $x^P$ and in the optimum $y = \boldsymbol{y}^T\boldsymbol{\lambda}$ and $b = \boldsymbol{b}^T\boldsymbol{\lambda}$ hold the model can be reformulated as

$$\begin{array}{lrcl} \max\limits_{\boldsymbol{\lambda}} & \frac{\boldsymbol{y}^T\boldsymbol{\lambda}}{\boldsymbol{b}^T\boldsymbol{\lambda}} & & \\ \text{s.t.} & x_i^{NP} & \geqq & \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} \\ & \boldsymbol{\lambda} & \geqq & \boldsymbol{0}. \end{array} \tag{A.8}$$

The corresponding materials balance model reads as

$$\begin{array}{lrcl} \max\limits_{y,b,x^P,\epsilon_{x^P},\epsilon_{x^{NP}},\epsilon_y,\epsilon_b,\boldsymbol{\lambda}} & \frac{y}{b} & & \\ \text{s.t.} & x^P & = & \boldsymbol{x}^{P^T}\boldsymbol{\lambda} + \epsilon_{x^P} \\ & x^{NP} & = & \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} + \epsilon_{x^{NP}} \\ & y & = & \boldsymbol{y}^T\boldsymbol{\lambda} - \epsilon_y \\ & b & = & \boldsymbol{b}^T\boldsymbol{\lambda} + \epsilon_b \\ & s_y\epsilon_y + s_{x^P}\epsilon_{x^P} & = & \epsilon_b \\ & y, b, x^P & \geqq & 0 \\ & \epsilon_{x^P}, \epsilon_y, \epsilon_b & \geqq & 0 \\ & \boldsymbol{\epsilon}_{\boldsymbol{x}^{NP}}, \boldsymbol{\lambda} & \geqq & \boldsymbol{0}. \end{array} \tag{A.9}$$

Since our good output electricity does not contain any materials $s_y = 0$ and $\epsilon_y$ can be removed from the programming problem. $\epsilon_{x^{NP}}$ can be removed since it does not affect the optimal choice of $y$ and $b$. Moreover, combining the restriction on the polluting input with the summing-up condition on the slacks leads to $\epsilon_b = s_{x^P}\left(x^P - \boldsymbol{x}^{P^T}\boldsymbol{\lambda}\right)$. Hence, the programming problem can be transformed into

$$\begin{array}{lrcl} \max\limits_{y,b,x^P,\epsilon_{x^P},\boldsymbol{\lambda}} & \frac{y}{b} & & \\ \text{s.t.} & x^P & = & \boldsymbol{x}^{P^T}\boldsymbol{\lambda} + \epsilon_{x^P} \\ & x_i^{NP} & \geqq & \boldsymbol{x}^{NP^T}\boldsymbol{\lambda} \\ & y & \leqq & \boldsymbol{y}^T\boldsymbol{\lambda} \\ & b & = & \boldsymbol{b}^T\boldsymbol{\lambda} + s_{x^P}\left(x^P - \boldsymbol{x}^{P^T}\boldsymbol{\lambda}\right) \\ & y, b, x^P, \epsilon_{x^P} & \geqq & 0 \\ & \boldsymbol{\lambda} & \geqq & \boldsymbol{0}. \end{array} \tag{A.10}$$

In the optimum $x^P - \boldsymbol{x}^{P^T}\boldsymbol{\lambda} = 0$, $y = \boldsymbol{y}^T\boldsymbol{\lambda}$ and $b = \boldsymbol{b}^T\boldsymbol{\lambda}$ hold. Therefore, this programming problem reduces to the programming problem of the joint production model.

## Appendix B

In this appendix we demonstrate how the optimal ratio $y/b$ can be estimated in a general model.

In the following optimization problem we assume that a DMU maximizes the ratio of a single good output (good output "o") to a single bad output (bad output "l") subject to a technology accounting for multiple polluting and non-polluting inputs, as well as multiple good and bad outputs. We start our discussion by assuming a joint production technology under constant returns to scale (CRS). In this case the optimal ratio for a DMU $i$ under evaluation can be obtained by solving the following fractional programming problem

$$\begin{aligned} \max_{y_o, b_l, \boldsymbol{\lambda}} \quad & \frac{y_o}{b_l} \\ \text{s.t.} \quad & \begin{bmatrix} \boldsymbol{x}_i^P \\ \boldsymbol{x}_i^{NP} \end{bmatrix} \geqq \begin{bmatrix} \boldsymbol{X}^P \\ \boldsymbol{X}^{NP} \end{bmatrix} \boldsymbol{\lambda} \\ & \begin{bmatrix} y_o \\ \boldsymbol{y}_{i,-o} \end{bmatrix} \leqq \begin{bmatrix} \boldsymbol{y}_o^T \\ \boldsymbol{Y}_{-o} \end{bmatrix} \boldsymbol{\lambda} \\ & \begin{bmatrix} b_l \\ \boldsymbol{b}_{i,-l} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_l^T \\ \boldsymbol{B}_{-l} \end{bmatrix} \boldsymbol{\lambda} \\ & y_o, b_l \geqq 0 \\ & \boldsymbol{\lambda} \geqq \mathbf{0}. \end{aligned} \tag{B.1}$$

In this program we have separated the inputs, good outputs and bad outputs in a suitable way for our following derivation. The input vector is partitioned into the polluting and the non-polluting inputs with $\boldsymbol{X}^P$ denoting the $m_1 \times n$ matrix of polluting inputs and $\boldsymbol{X}^{NP}$ the $m_2 \times n$ matrix of non-polluting inputs. The good outputs are partitioned into the single good output which is a part of the objective function ($y_o$) and the remaining $(k-1)$ good outputs which are fixed $(\boldsymbol{y}_{i,-o})$, with $\boldsymbol{y}_o^T$ representing the transpose of the $n \times 1$ vector of good output "o" and $\boldsymbol{Y}_{-o}$ denoting the $(k-1) \times n$ matrix of the remaining good outputs. Similarly, the bad outputs are partitioned into the single pollutant which is part of the objective function ($b_l$) and the remaining $(s-1)$ bad outputs which are fixed ($\boldsymbol{b}_{i,-l}$) with $\boldsymbol{b}_l^T$ representing the transpose of the $n \times 1$ vector of bad output "l" and $\boldsymbol{B}_{-l}$ denoting the $(s-1) \times n$ matrix of the remaining bad outputs.

To linearize programming problem (B.1) note that $y_o = \boldsymbol{y}_o^T \boldsymbol{\lambda}$ in the optimum. Otherwise the obtained ratio cannot be maximal. Moreover, by the weak disposability assumption $b_l = \boldsymbol{b}_l^T \boldsymbol{\lambda}$ holds as well. By inserting these constraints in the objective function the program can be reformulated as

$$\begin{aligned} \max_{\boldsymbol{\lambda}} \quad & \frac{\boldsymbol{y}_o^T \boldsymbol{\lambda}}{\boldsymbol{b}_l^T \boldsymbol{\lambda}} \\ \text{s.t.} \quad & \begin{bmatrix} \boldsymbol{x}_i^P \\ \boldsymbol{x}_i^{NP} \end{bmatrix} \geqq \begin{bmatrix} \boldsymbol{X}^P \\ \boldsymbol{X}^{NP} \end{bmatrix} \boldsymbol{\lambda} \\ & \boldsymbol{y}_{i,-o} \leqq \boldsymbol{Y}_{-o} \boldsymbol{\lambda} \\ & \boldsymbol{b}_{i,-l} = \boldsymbol{B}_{-l} \boldsymbol{\lambda} \\ & \boldsymbol{\lambda} \geqq \mathbf{0}. \end{aligned} \tag{B.2}$$

Denoting $w = \frac{1}{\boldsymbol{b}_l^T \boldsymbol{\lambda}}$ and $\boldsymbol{z} = \frac{1}{\boldsymbol{b}_l^T \boldsymbol{\lambda}} \boldsymbol{\lambda}$ and applying the transformation by Charnes and Cooper (1962) this fractional optimization problem can be transformed into the linear programming problem

$$\begin{aligned}
\max_{w,\boldsymbol{z}} \quad & \boldsymbol{y}_o^T \boldsymbol{z} \\
\text{s.t.} \quad & \begin{bmatrix} w\boldsymbol{x}_i^P \\ w\boldsymbol{x}_i^{NP} \end{bmatrix} \geqq \begin{bmatrix} \boldsymbol{X}^P \\ \boldsymbol{X}^{NP} \end{bmatrix} \boldsymbol{z} \\
& w\boldsymbol{y}_{i,-o} \leqq \boldsymbol{Y}_{-o}\boldsymbol{z} \\
& w\boldsymbol{b}_{i,-l} = \boldsymbol{B}_{-l}\boldsymbol{z} \\
& \boldsymbol{b}_l^T \boldsymbol{z} = 1 \\
& w \geqq 0 \\
& \boldsymbol{z} \geqq \boldsymbol{0}.
\end{aligned} \tag{B.3}$$

The VRS counterpart to (B.1) is defined by:

$$\begin{aligned}
\max_{y_o, b_l, \boldsymbol{\lambda}, \theta} \quad & \frac{y_o}{b_l} \\
\text{s.t.} \quad & \begin{bmatrix} \boldsymbol{x}_i^P \\ \boldsymbol{x}_i^{NP} \end{bmatrix} \geqq \begin{bmatrix} \boldsymbol{X}^P \\ \boldsymbol{X}^{NP} \end{bmatrix} \boldsymbol{\lambda} \\
& \begin{bmatrix} \boldsymbol{y}_o \\ \boldsymbol{y}_{i,-o} \end{bmatrix} \leqq \begin{bmatrix} \boldsymbol{y}_o^T \\ \boldsymbol{Y}_{-o} \end{bmatrix} \boldsymbol{\lambda}\theta \\
& \begin{bmatrix} \boldsymbol{b}_l \\ \boldsymbol{b}_{i,-l} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_l^T \\ \boldsymbol{B}_{-l} \end{bmatrix} \boldsymbol{\lambda}\theta \\
& y_o, b_l \geqq 0 \\
& \boldsymbol{\lambda} \geqq \boldsymbol{0} \\
& 0 \leqq \theta \leqq 1.
\end{aligned} \tag{B.4}$$

where $\theta$ denotes the scaling factor by the weak disposability assumption and is an endogenous variable. Program (B.4) can be transformed by dividing the good and the bad output constraints by $\theta$. Denoting $\rho = 1/\theta$ the resulting maximization problem reads as

$$\begin{aligned}
\max_{y_o, b_l, \boldsymbol{\lambda}, \rho} \quad & \frac{y_o}{b_l} \\
\text{s.t.} \quad & \begin{bmatrix} \boldsymbol{x}_i^P \\ \boldsymbol{x}_i^{NP} \end{bmatrix} \geqq \begin{bmatrix} \boldsymbol{X}^P \\ \boldsymbol{X}^{NP} \end{bmatrix} \boldsymbol{\lambda} \\
& \begin{bmatrix} \rho y_o \\ \rho \boldsymbol{y}_{i,-o} \end{bmatrix} \leqq \begin{bmatrix} \boldsymbol{y}_o^T \\ \boldsymbol{Y}_{-o} \end{bmatrix} \boldsymbol{\lambda} \\
& \begin{bmatrix} \rho b_l \\ \rho \boldsymbol{b}_{i,-l} \end{bmatrix} = \begin{bmatrix} \boldsymbol{b}_l^T \\ \boldsymbol{B}_{-l} \end{bmatrix} \boldsymbol{\lambda} \\
& y_o, b_l \geqq 0 \\
& \boldsymbol{\lambda} \geqq \boldsymbol{0} \\
& \rho \geqq 1.
\end{aligned} \tag{B.5}$$

Dividing each constraint by $\rho$ leads to

$$\begin{aligned}
\max_{y_o, b_l, \boldsymbol{\lambda}, \rho} \quad & \frac{y_o}{b_l} \\
\text{s.t.} \quad \begin{bmatrix} \frac{\boldsymbol{x}_i^P}{\rho} \\ \frac{\boldsymbol{x}_i^{NP}}{\rho} \end{bmatrix} & \geqq \begin{bmatrix} \boldsymbol{X}^P \\ \boldsymbol{X}^{NP} \end{bmatrix} \frac{\boldsymbol{\lambda}}{\rho} \\
\begin{bmatrix} y_o \\ \boldsymbol{y}_{i,-o} \end{bmatrix} & \leqq \begin{bmatrix} \boldsymbol{y}_o^T \\ \boldsymbol{Y}_{-o} \end{bmatrix} \frac{\boldsymbol{\lambda}}{\rho} \\
\begin{bmatrix} b_l \\ \boldsymbol{b}_{i,-l} \end{bmatrix} & = \begin{bmatrix} \boldsymbol{b}_l^T \\ \boldsymbol{B}_{-l} \end{bmatrix} \frac{\boldsymbol{\lambda}}{\rho} \\
y_o, b_l & \geqq 0 \\
\frac{\boldsymbol{\lambda}}{\rho} & \geqq \boldsymbol{0} \\
\boldsymbol{1}^T \frac{\boldsymbol{\lambda}}{\rho} & = \frac{1}{\rho} \\
1 & \geqq \frac{1}{\rho}.
\end{aligned} \tag{B.6}$$

Denoting $\frac{\boldsymbol{\lambda}}{\rho} = \boldsymbol{\mu}$ and replacing $\frac{1}{\rho}$ by $\boldsymbol{1}^T\boldsymbol{\mu}$ the programming problem can be reformulated as

$$\begin{aligned}
\max_{y_o, b_l, \boldsymbol{\mu}} \quad & \frac{y_o}{b_l} \\
\text{s.t.} \quad \begin{bmatrix} \boldsymbol{1}^T\boldsymbol{\mu}\boldsymbol{x}_i^P \\ \boldsymbol{1}^T\boldsymbol{\mu}\boldsymbol{x}_i^{NP} \end{bmatrix} & \geqq \begin{bmatrix} \boldsymbol{X}^P \\ \boldsymbol{X}^{NP} \end{bmatrix} \boldsymbol{\mu} \\
\begin{bmatrix} \boldsymbol{y}_o \\ \boldsymbol{y}_{i,-o} \end{bmatrix} & \leqq \begin{bmatrix} \boldsymbol{y}_o^T \\ \boldsymbol{Y}_{-o} \end{bmatrix} \boldsymbol{\mu} \\
\begin{bmatrix} \boldsymbol{b}_l \\ \boldsymbol{b}_{i,-l} \end{bmatrix} & = \begin{bmatrix} \boldsymbol{b}_l^T \\ \boldsymbol{B}_{-l} \end{bmatrix} \boldsymbol{\mu} \\
y_o, b_l & \geqq 0 \\
\boldsymbol{\mu} & \geqq \boldsymbol{0} \\
\boldsymbol{1}^T\boldsymbol{\mu} & \leqq 1.
\end{aligned} \tag{B.7}$$

As in case of the analysis under CRS the optimization variables in the objective function are replaced by the associated constraints since again these constraints hold with equality in the optimum. Moreover, the constraints on the inputs are slightly rearranged leading to

$$\begin{aligned}
\max_{\boldsymbol{\mu}} \quad & \frac{\boldsymbol{y}_o^T\boldsymbol{\mu}}{\boldsymbol{b}_l^T\boldsymbol{\mu}} \\
\text{s.t.} \quad \begin{bmatrix} \boldsymbol{X}^P - \boldsymbol{x}_i^P \\ \boldsymbol{X}^{NP} - \boldsymbol{x}_i^{NP} \end{bmatrix} \boldsymbol{\mu} & \geqq \boldsymbol{0} \\
\boldsymbol{y}_{i,-o} & \leqq \boldsymbol{Y}_{-o}\boldsymbol{\mu} \\
\boldsymbol{b}_{i,-l} & = \boldsymbol{B}_{-l}\boldsymbol{\mu} \\
\boldsymbol{\mu} & \geqq \boldsymbol{0} \\
\boldsymbol{1}^T\boldsymbol{\mu} & \leqq 1.
\end{aligned} \tag{B.8}$$

Applying the Charnes-Cooper transformation and denoting $h = \frac{1}{\boldsymbol{b}_l^T\boldsymbol{\mu}}$ and $\boldsymbol{g} = \frac{1}{\boldsymbol{b}_l^T\boldsymbol{\mu}}\boldsymbol{\mu}$ leads to the linear programming problem

$$
\begin{array}{rcl}
\max\limits_{h,\boldsymbol{g}} & & \boldsymbol{y}_o^T\boldsymbol{g} \\
\text{s.t.} \begin{bmatrix} \boldsymbol{X}^P - \boldsymbol{x}_i^P \\ \boldsymbol{X}^{NP} - \boldsymbol{x}_i^{NP} \end{bmatrix}\boldsymbol{g} & \leqq & \boldsymbol{0} \\
h\boldsymbol{y}_{i,-o} & \leqq & \boldsymbol{Y}_{-o}\boldsymbol{g} \\
h\boldsymbol{b}_{i,-l} & = & \boldsymbol{B}_{-l}\boldsymbol{g} \\
\boldsymbol{b}_l^T\boldsymbol{g} & = & 1 \\
h & \geqq & 0 \\
\boldsymbol{g} & \geqq & \boldsymbol{0}.
\end{array}
\tag{B.9}
$$

The MB model under CRS reads as[25]

$$
\begin{array}{rcl}
\max\limits_{y_o,b_l,\boldsymbol{\epsilon}_{\boldsymbol{x}^P},\boldsymbol{\epsilon}_{\boldsymbol{x}^{NP}},\epsilon_{y_o},\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}},\epsilon_{b_l},\boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}},\boldsymbol{\lambda}} & & \dfrac{y_o}{b_l} \\
\text{s.t.} \begin{bmatrix} \boldsymbol{x}_i^P \\ \boldsymbol{x}_i^{NP} \end{bmatrix} & = & \begin{bmatrix} \boldsymbol{X}^P\boldsymbol{\lambda} + \boldsymbol{\epsilon}_{\boldsymbol{x}^P} \\ \boldsymbol{X}^{NP}\boldsymbol{\lambda} + \boldsymbol{\epsilon}_{\boldsymbol{x}^{NP}} \end{bmatrix} \\
\begin{bmatrix} y_o \\ \boldsymbol{y}_{i,-o} \end{bmatrix} & = & \begin{bmatrix} \boldsymbol{y}_o^T\boldsymbol{\lambda} - \epsilon_{y_o} \\ \boldsymbol{Y}_{-o}\boldsymbol{\lambda} - \boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}} \end{bmatrix} \\
\begin{bmatrix} b_l \\ \boldsymbol{b}_{i,-l} \end{bmatrix} & = & \begin{bmatrix} \boldsymbol{b}_l^T\boldsymbol{\lambda} + \epsilon_{b_l} \\ \boldsymbol{B}_{-l}\boldsymbol{\lambda} + \boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}} \end{bmatrix} \\
\begin{bmatrix} \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\boldsymbol{\epsilon}_{\boldsymbol{x}^P} + \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}} + s_{y_o,b_l}\epsilon_{y_o} \\ \boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}}\boldsymbol{\epsilon}_{\boldsymbol{x}^P} + \boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}}\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}} + \boldsymbol{s}_{y_o,\boldsymbol{b}_{-l}}\epsilon_{y_o} \end{bmatrix} & = & \begin{bmatrix} \epsilon_{b_l} \\ \boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}} \end{bmatrix} \\
y_o, b_l & \geqq & 0 \\
\epsilon_{y_o}, \epsilon_{b_l} & \geqq & 0 \\
\boldsymbol{\epsilon}_{\boldsymbol{x}^P}, \boldsymbol{\epsilon}_{\boldsymbol{x}^{NP}}, \boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}}, \boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}}, \boldsymbol{\lambda} & \geqq & \boldsymbol{0}.
\end{array}
\tag{B.10}
$$

As in case of the joint production model we have separated the constraints for the good outputs and the bad outputs. In addition to the variables defined above we denote $\boldsymbol{\epsilon}_{\boldsymbol{x}^P}$ ($\boldsymbol{\epsilon}_{\boldsymbol{x}^{NP}}$) the $m_1 \times 1$ ($m_2 \times 1$) vector of slacks for the polluting (non-polluting) inputs. The scalar $\boldsymbol{\epsilon}_{y_o}$ ($\epsilon_{b_l}$)denotes the slack for the good output "o" (the bad output "l") while $\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}}$ ($\boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}}$) denotes the $(k-1) \times 1$ ($(s-1) \times 1$) vector of the slacks for the remaining good (bad) outputs. By the weak G-disposability axiom (MB8) the slacks for the bad outputs are linear functions of the slacks for the polluting inputs and the good outputs. In the constraint for the slack variable of the bad output "l" $\boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T$ represents the transpose of the $m_1 \times 1$ vector of emission factors of the polluting inputs, $\boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T$ denotes the transpose of the $(k-1) \times 1$ vector of recuperation factors for all good outputs except for the output "o", and $s_{y_o,b_l}$ denotes the scalar recuperation factor for the good output "o". In the constraint for the slacks of the remaining bad outputs $\boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}}$ denotes the $(s-1) \times m_1$ matrix of emission factors for the polluting inputs, $\boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}}$ represents the $(s-1) \times (k-1)$ matrix of recuperation factors for the good outputs without good output "o", and $\boldsymbol{s}_{y_o,\boldsymbol{b}_{-l}}$ denotes the $(s-1) \times 1$ vector of recuperation factors for the good output "o".

*Linearization of (B.10)*

Inserting the equality constraints on the good output "o" and the bad output "l" in the objective

[25] Note that this transformation can be demonstrated similarly for the case of variable returns to scale by adding the constraint $\boldsymbol{1}^T\boldsymbol{\lambda} = 1$.

function of (B.10) leads to

$$\begin{aligned}
\max_{\boldsymbol{\epsilon}_{\boldsymbol{x}^P},\boldsymbol{\epsilon}_{\boldsymbol{x}^{NP}},\epsilon_{y_o},\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}},\epsilon_{b_l},\boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}},\boldsymbol{\lambda}} \frac{\boldsymbol{y}_o^T\boldsymbol{\lambda}-\epsilon_{y_o}}{\boldsymbol{b}_l^T\boldsymbol{\lambda}+\epsilon_{b_l}} & \\
\text{s.t.} \begin{bmatrix}\boldsymbol{x}_i^P\\ \boldsymbol{x}_i^{NP}\end{bmatrix} &= \begin{bmatrix}\boldsymbol{X}^P\boldsymbol{\lambda}+\boldsymbol{\epsilon}_{\boldsymbol{x}^P}\\ \boldsymbol{X}^{NP}\boldsymbol{\lambda}+\boldsymbol{\epsilon}_{\boldsymbol{x}^{NP}}\end{bmatrix} \\
\boldsymbol{y}_{i,-o} &= \boldsymbol{Y}_{-o}\boldsymbol{\lambda}-\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}} \\
\boldsymbol{b}_{i,-l} &= \boldsymbol{B}_{-l}\boldsymbol{\lambda}+\boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}} \\
\begin{bmatrix}\boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\boldsymbol{\epsilon}_{\boldsymbol{x}^P}+\boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}}+s_{y_o,b_l}\epsilon_{y_o}\\ \boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}}\boldsymbol{\epsilon}_{\boldsymbol{x}^P}+\boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}}\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}}+\boldsymbol{s}_{y_o,\boldsymbol{b}_{-l}}\epsilon_{y_o}\end{bmatrix} &= \begin{bmatrix}\epsilon_{b_l}\\ \boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}}\end{bmatrix} \\
\epsilon_{y_o},\epsilon_{b_l} &\geqq 0 \\
\boldsymbol{\epsilon}_{\boldsymbol{x}^P},\boldsymbol{\epsilon}_{\boldsymbol{x}^{NP}},\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}},\boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}},\boldsymbol{\lambda} &\geqq \mathbf{0}.
\end{aligned} \tag{B.11}$$

The constraints for the polluting and non-polluting inputs as well as the good outputs and the bad outputs can be rearranged to

$$\begin{aligned}
\boldsymbol{\epsilon}_{\boldsymbol{x}^P} &= \boldsymbol{x}_i^P-\boldsymbol{X}^P\boldsymbol{\lambda} \\
\boldsymbol{\epsilon}_{\boldsymbol{x}^{NP}} &= \boldsymbol{x}_i^{NP}-\boldsymbol{X}^{NP}\boldsymbol{\lambda} \\
\boldsymbol{\epsilon}_{\boldsymbol{y}_{-o}} &= \boldsymbol{Y}_{-o}\boldsymbol{\lambda}-\boldsymbol{y}_{i,-o} \\
\boldsymbol{\epsilon}_{\boldsymbol{b}_{-l}} &= \boldsymbol{b}_{i,-l}-\boldsymbol{B}_{-l}\boldsymbol{\lambda}
\end{aligned} \tag{B.12}$$

Replacing these expressions in the constraints on the slacks for the bad outputs and the non-negativity constraints we obtain the programming problem

$$\begin{aligned}
\max_{\epsilon_{y_o},\epsilon_{b_l},\boldsymbol{\lambda}} \frac{\boldsymbol{y}_o^T\boldsymbol{\lambda}-\epsilon_{y_o}}{\boldsymbol{b}_l^T\boldsymbol{\lambda}+\epsilon_{b_l}} & \\
\text{s.t.} \begin{bmatrix}\boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\left(\boldsymbol{x}_i^P-\boldsymbol{X}^P\boldsymbol{\lambda}\right)+\boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T\left(\boldsymbol{Y}_{-o}\boldsymbol{\lambda}-\boldsymbol{y}_{i,-o}\right)\\ +s_{y_o,b_l}\epsilon_{y_o}\\ \boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}}\left(\boldsymbol{x}_i^P-\boldsymbol{X}^P\boldsymbol{\lambda}\right)+\boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}}\left(\boldsymbol{Y}_{-o}\boldsymbol{\lambda}-\boldsymbol{y}_{i,-o}\right)\\ +\boldsymbol{s}_{y_o,\boldsymbol{b}_{-l}}\epsilon_{y_o}\end{bmatrix} &= \begin{bmatrix}\epsilon_{b_l}\\ \boldsymbol{b}_{i,-l}-\boldsymbol{B}_{-l}\boldsymbol{\lambda}\end{bmatrix} \\
\begin{bmatrix}\boldsymbol{x}_i^P\\ \boldsymbol{x}_i^{NP}\end{bmatrix} &\geqq \begin{bmatrix}\boldsymbol{X}^P\boldsymbol{\lambda}\\ \boldsymbol{X}^{NP}\boldsymbol{\lambda}\end{bmatrix} \\
\boldsymbol{y}_{i,-o} &\leqq \boldsymbol{Y}_{-o}\boldsymbol{\lambda} \\
\boldsymbol{b}_{i,-l} &\geqq \boldsymbol{B}_{-l}\boldsymbol{\lambda} \\
\epsilon_{y_o},\epsilon_{b_l} &\geqq 0 \\
\boldsymbol{\lambda} &\geqq \mathbf{0}.
\end{aligned} \tag{B.13}$$

Inserting the constraint on the slack of bad output "l" in the objective function and in the non-negativity constraint as well as combining the equality constraint on $\boldsymbol{b}_{i,-l}-\boldsymbol{B}_{-l}\boldsymbol{\lambda}$ with the respective inequality constraint leads to

$$
\begin{aligned}
\max_{\epsilon_{y_o},\boldsymbol{\lambda}} & \frac{\boldsymbol{y}_o^T\boldsymbol{\lambda} - \epsilon_{y_o}}{\boldsymbol{b}_l^T\boldsymbol{\lambda} + \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\left(\boldsymbol{x}_i^P - \boldsymbol{X}^P\boldsymbol{\lambda}\right) + \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T\left(\boldsymbol{Y}_{-o}\boldsymbol{\lambda} - \boldsymbol{y}_{i,-o}\right) + s_{y_o,b_l}\epsilon_{y_o}} \\
\text{s.t.} \begin{bmatrix}\boldsymbol{x}_i^P \\ \boldsymbol{x}_i^{NP}\end{bmatrix} &\geqq \begin{bmatrix}\boldsymbol{X}^P\boldsymbol{\lambda} \\ \boldsymbol{X}^{NP}\boldsymbol{\lambda}\end{bmatrix} \\
\boldsymbol{y}_{i,-o} &\leqq \boldsymbol{Y}_{-o}\boldsymbol{\lambda} \\
\boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}}\boldsymbol{x}_i^P - \boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}}\boldsymbol{y}_{i,-o} &\geqq \boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}}\boldsymbol{X}^P\boldsymbol{\lambda} - \boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}}\boldsymbol{Y}_{-o}\boldsymbol{\lambda} - \boldsymbol{s}_{y_o,\boldsymbol{b}_{-l}}\epsilon_{y_o} \\
\boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\boldsymbol{x}_i^P - \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}\boldsymbol{y}_{i,-o} &\geqq \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\boldsymbol{X}^P\boldsymbol{\lambda} - \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T\boldsymbol{Y}_{-o}\boldsymbol{\lambda} - s_{y_o,b_l}\epsilon_{y_o} \\
\epsilon_{y_o} &\geqq 0 \\
\boldsymbol{\lambda} &\geqq \boldsymbol{0}.
\end{aligned}
\tag{B.14}
$$

By defining the new vector $\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix}$ the notation in the programming problem can be slightly changed to

$$
\begin{aligned}
\max_{\epsilon_{y_o},\boldsymbol{\lambda}} & \frac{\begin{pmatrix}\boldsymbol{y}_o^T \\ -1\end{pmatrix}^T\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix}}{\begin{pmatrix}\boldsymbol{b}_l^T - \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\boldsymbol{X}^P + \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T\boldsymbol{Y}_{-o} \\ s_{y_o,b_l}\end{pmatrix}^T\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} + \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\boldsymbol{x}_i^P - \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T\boldsymbol{y}_{i,-o}} \\
\text{s.t.} \begin{bmatrix}\boldsymbol{x}_i^P \\ \boldsymbol{x}_i^{NP}\end{bmatrix} &\geqq \begin{bmatrix}\begin{pmatrix}\boldsymbol{X}^P & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} \\ \begin{pmatrix}\boldsymbol{X}^{NP} & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix}\end{bmatrix} \\
\boldsymbol{y}_{i,-o} &\leqq \begin{pmatrix}\boldsymbol{Y}_{-o} & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} \\
\boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}}\boldsymbol{x}_i^P - \boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}}\boldsymbol{y}_{i,-o} &\geqq \boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}}\begin{pmatrix}\boldsymbol{X}^P & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} \\
&\quad -\boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}}\begin{pmatrix}\boldsymbol{Y}_{-o} & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} \\
&\quad -\begin{pmatrix}\boldsymbol{0} & \boldsymbol{s}_{y_o,\boldsymbol{b}_{-l}}\end{pmatrix}\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} \\
\boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\boldsymbol{x}_i^P - \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T\boldsymbol{y}_{i,-o} &\geqq \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T\begin{pmatrix}\boldsymbol{X}^P & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} \\
&\quad -\boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T\begin{pmatrix}\boldsymbol{Y}_{-o} & \boldsymbol{0}\end{pmatrix}\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} \\
&\quad -\begin{pmatrix}\boldsymbol{0} & s_{y_o,b_l}\end{pmatrix}\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} \\
\begin{pmatrix}\boldsymbol{\lambda} \\ \epsilon_{y_o}\end{pmatrix} &\geqq \boldsymbol{0}.
\end{aligned}
\tag{B.15}
$$

Defining the new variables

$$
\begin{aligned}
d &= \frac{1}{\begin{pmatrix} \boldsymbol{b}_l^T - \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T \boldsymbol{X}^P + \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T \boldsymbol{Y}_{-o} \\ s_{y_o,b_l} \end{pmatrix}^T \begin{pmatrix} \boldsymbol{\lambda} \\ \epsilon_{y_o} \end{pmatrix} + \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T \boldsymbol{x}_i^P - \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T \boldsymbol{y}_{i,-o}} \\
\boldsymbol{f} &= \frac{1}{\begin{pmatrix} \boldsymbol{b}_l^T - \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T \boldsymbol{X}^P + \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T \boldsymbol{Y}_{-o} \\ s_{y_o,b_l} \end{pmatrix}^T \begin{pmatrix} \boldsymbol{\lambda} \\ \epsilon_{y_o} \end{pmatrix} + \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T \boldsymbol{x}_i^P - \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T \boldsymbol{y}_{i,-o}} \begin{pmatrix} \boldsymbol{\lambda} \\ \epsilon_{y_o} \end{pmatrix}
\end{aligned}
\tag{B.16}
$$

and applying the Charnes-Cooper transformation leads to the linearized programming problem

$$
\begin{aligned}
\max_{d,\boldsymbol{f}} \begin{pmatrix} \boldsymbol{y}_o \\ -1 \end{pmatrix} \boldsymbol{f} & \\
\text{s.t.} \begin{bmatrix} d\boldsymbol{x}_i^P \\ d\boldsymbol{x}_i^{NP} \end{bmatrix} &\geqq \begin{bmatrix} \begin{pmatrix} \boldsymbol{X}^P & \boldsymbol{0} \end{pmatrix} \boldsymbol{f} \\ \begin{pmatrix} \boldsymbol{X}^{NP} & \boldsymbol{0} \end{pmatrix} \boldsymbol{f} \end{bmatrix} \\
d\boldsymbol{y}_{i,-o} &\leqq \begin{pmatrix} \boldsymbol{Y}_{-o} & \boldsymbol{0} \end{pmatrix} \boldsymbol{z} \\
d\left( \boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}} \boldsymbol{x}_i^P - \boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}} \boldsymbol{y}_{i,-o} \right) &\geqq \Big( \boldsymbol{S}_{\boldsymbol{x}^P,\boldsymbol{b}_{-l}} \begin{pmatrix} \boldsymbol{X}^P & \boldsymbol{0} \end{pmatrix} \\
&\quad - \boldsymbol{S}_{\boldsymbol{y}_{-o},\boldsymbol{b}_{-l}} \begin{pmatrix} \boldsymbol{Y}_{-o} & \boldsymbol{0} \end{pmatrix} \\
&\quad - \begin{pmatrix} \boldsymbol{0} & \boldsymbol{s}_{y_o,\boldsymbol{b}_{-l}} \end{pmatrix} \Big) \boldsymbol{f} \\
d\left( \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T \boldsymbol{x}_i^P - \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T \boldsymbol{y}_{i,-o} \right) &\geqq \Big( \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T \begin{pmatrix} \boldsymbol{X}^P & \boldsymbol{0} \end{pmatrix} \\
&\quad - \boldsymbol{s}_{\boldsymbol{y}_{-o},b_l}^T \begin{pmatrix} \boldsymbol{Y}_{-o} & \boldsymbol{0} \end{pmatrix} \\
&\quad - \begin{pmatrix} \boldsymbol{0} & s_{y_o,b_l} \end{pmatrix} \Big) \boldsymbol{f} \\
\begin{pmatrix} \boldsymbol{b}_o - \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T \boldsymbol{X}^P + \boldsymbol{s}_{\boldsymbol{y}_{-o,b_l}}^T \boldsymbol{Y}_{-o} \\ s_{y_o,b_l} \end{pmatrix}^T \boldsymbol{f} & \\
+ d\left( \boldsymbol{s}_{\boldsymbol{x}^P,b_l}^T \boldsymbol{x}_i^P - \boldsymbol{s}_{\boldsymbol{y}_{-o,b_l}}^T \boldsymbol{y}_{i,-o} \right) &= 1 \\
d &\geqq 0 \\
\boldsymbol{f} &\geqq 0.
\end{aligned}
\tag{B.17}
$$