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Project selection with sets of mutually exclusive alternatives

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1 Introduction

We study the problem to maximise the net economic benefit of an investment plan by selecting from a portfolio of candidate projects within a given budget constraint. One example would be the national transport plans in countries like Norway and Sweden. Assuming independent projects, i.e. (1) all projects may be selected regardless of which other projects are selected, and (2) their benefits and costs stay the same regardless of which other projects are selected, the economic efficiency of the entire investment plan is maximised if projects are selected according to their benefit-cost ratio until the budget is exhausted. To be exact, this result requires projects to be infinitely divisible, but the divisibility matters only for the last project to be included in the plan, and so is of little consequence if projects are small compared to the budget.

Normally, however, the planning of a project involves a stage where a set of alternative concepts or designs are considered. A best alternative is chosen, and the plan is composed from the pool of all such best alternative solutions. This two-step procedure violates the assumptions underlying the benefit-cost ratio criterion, and in fact, neither the benefit-cost ratio nor the net present value of a project is a valid choice criterion in this case.

In this paper, we set out the correct criterion to use in this case. It is not the first time this criterion had been proposed. Actually, it was proposed as early as 1955 by Lorie and Savage, but even if it was commented upon by authors such as Weingartner (1963, 1966) and others in the sixties, it obviously got lost in the subsequent more and more complex development of the capital budgeting literature. We show that the criterion is the solution to a one-period continuous knapsack problem with mutually exclusive project alternatives, and that an approximate solution can be found by a simple iterative procedure, just like Lorie and Savage said.

Section 2 prepares for the derivation of the Lorie and Savage criterion in section 3. This it does by reminding the reader of how the benefit cost criterion is derived: It is the solution to a linear programming problem called the continuous knapsack problem with independent projects. The assumptions underlying this problem are necessary and sufficient conditions for the benefit cost criterion to be valid. Changing the assumption of independent projects to projects with mutually exclusive alternatives must produce a different criterion, namely the Lorie and Savage criterion, as shown in section 3. In section 4, we illustrate the way this criterion functions in a real life example from Norwegian transport planning. In section 5, we briefly discuss the situations when the new criterion might be of use and its implication for the possibility of local decisions. Section 6 concludes.

2 The benefit cost ratio

Judging from the HEATCO survey of how cost benefit analysis is practised in 25 European countries (HEATCO 2005a and b), some confusion still exists about the definition of costs to be used in the benefit cost ratio, about its relationship to the net present value and other commonly used indicators, and about the conditions for its validity as a decision-making tool. Even the HEATCO recommendations themselves

(HEATCO 2006) are plainly wrong when they define costs (to be entered in the denominator of the ratio) as the resource consumption of transport providers and government, and benefits (to be entered in the nominator) as the resource gains of travellers and third parties. This is shown in this section. We also show the necessary and sufficient conditions for the benefit cost ratio to be a valid criterion for project selection.

Nearly all of the countries surveyed in HEATCO report that they combine the benefit cost ratio and the net present value. Many of them provide a clear description of when to use the one or the other, but there seem to be some that use some undefined mix of them. Furthermore, fairly many countries use the internal rate of return to compare projects (a criterion that is not suitable for comparing mutually exclusive options, and that may produce wrong results unless all costs occur before all benefits), or even the payback period (a practise that does not take all relevant costs and benefits into consideration).

Assume that our objective is to maximise the net present value of a plan within a given budget constraint. The candidate projects are assumed to be infinitely divisible and mutually independent. That is, any fraction of the costs of a given project will produce a similar fraction of the benefits, and the costs and benefits of a candidate project is not at all dependent on which of the other projects that are included in the plan. There are no other objectives than maximisation of net present value, and no constraints or conditions other than the given budget constraint. We want to show that the necessary and sufficient condition to achieve our objective under these circumstances is that we select projects in descending order of their benefit cost ratio (with costs defined as net outlays over the relevant public budget) until the budget is exhausted. To keep within the budget, only a fraction of the last selected project can normally be implemented.

2.1 The solution to a linear programming problem

Let $\boldsymbol{b} = (b_1,\ldots,b_n)$ be the net present benefit of n candidate projects, some of which are to be chosen to form the plan of a government agency. Let $\boldsymbol{c} = (c_1,\ldots,c_n)$ be the vector of discounted net payments that the agency must incur if these candidate projects are to be included in the plan. We assume there is a constraint a on the net present value of the agency's budgets in the period we consider.

The assumption of such a constraint seems to contradict one of the implicit assumptions of discounting, namely free lending and loaning at the same interest rate. The contradiction is resolved if we assume that the constraint is imposed by a political decision at a higher level of government, as it usually is. Such a decision may make sense even if the margin between the lending and loan rate for the government is very small, because the agency's spending involves not just money, but real resources in short supply.

The n prosjects are infinitely divisible. That is, if we carry out only a part of a project, as measured by budget outlays, we will always achieve the same part of the project's net benefits. This is certainly not always reasonable, but it matters less and less the smaller the projects are as parts of the budget. Let $\boldsymbol{x} = (x_1,\ldots,x_n)$ be the parts of each of the projects that are implemented. Thus $x_j \in [0,1]$ for all x_j. Finally, we assume that all projects are independent of each other, i.e., no element of $\boldsymbol{b}$ and $\boldsymbol{c}$ are functions of $\boldsymbol{x}$. If this seems to be a problematic assumption in any given case, it can often be solved by forming all possible combinations of the interdependent projects and enter these combinations instead of the interdependent projects themselves. But what we have then are mutually exclusive alternatives, and the rule of section 3 must be applied.

Projects that do not require any part of the budget can be decided upon separately, and projects with negative net benefits should always be discarded. Thus we may assume without problems that all elements of ***b*** and ***c*** are strictly positive and that all elements of ***b*** are larger than or equal to their corresponding element of ***c***.

The linear programming problem (LP1) based on these assumptions can now be formulated. Implicitly, it is also assumed that there are no binding restrictions other than the budget on the selection of projects. For example, there is no quantified target for the reduction of climate gas emissions.

$$\text{(LP1)} \qquad \max_{\mathbf{x}} \ \sum_{j=1}^{n} b_j x_j \quad \text{s.t.} \ \sum_{j=1}^{n} \mathrm{c}_j x_j \le a \ \text{ og } \ x_j \in [0,1] \ \ \forall j$$

The solution to the problem (LP1) is to arrange the candidate projects after their cost benefit ratio b_j/c_j and select them from the top until the budget is used up. Say that the candidate projects are numbered so that $b_1/c_1 \ge b_2/c_2 \ge \ldots \ge b_n/c_n$. If we select them in the order 1, 2, 3, … and so on, we will ultimately come to a project number r such that the sum of the $r - 1$ first costs c is less than the budget a, while the sum of the r first is greater than a. Formally, the solution can be written as Equation (1) on the next page.

The formal proof that (1) is indeed the solution requires use of the Simplex method, see any textbook in linear programming. An intuitive argument is this: Assume, contrary to (1), that the solution is to exclude some project with a higher benefit cost ratio b_j/c_j than at least one of the r projects selected by (1). If we take out a small slice of project r and replace it by a similar slice of this excluded project, the objective function must increase. Thus in the the optimal solution, all selected projects must have higher benefit cost ratios than any project not selected.

$$\text{(1)} \qquad x_j = \begin{cases} 1 & \text{for } j = 1,\ldots,r-1 \\ \dfrac{a - \sum_{j=1}^{r-1} c_j}{c_r} & \text{for } j = r \\ 0 & \text{for } j = r+1,\ldots,n \end{cases}$$

Diagram 1 illustrates our finding. There, all projects are ordered by the benefit cost ratio (BCR) and entered in the diagram as columns of different height and width. The width of a column is its cost, c_j, and the height is the benefit cost ratio b_j/c_j. Since c_j*(b_j/c_j) = b_j, the area of column *j* represents the net present value of project *j*. The vertical line *a* represents the budget. The area of all columns to the left of *a* is the net benefit of all projects financed within the budget. It is seen that the only project that have to be divided is project 6. On the right side of the line *a* are the projects that are excluded from the plan. We have just concluded that the gros benefit of the plan is maximised if projects are selected according to their BCR. Since the cost of the plan always equals the constant *a*, this strategy also maximises the net present value of the plan.

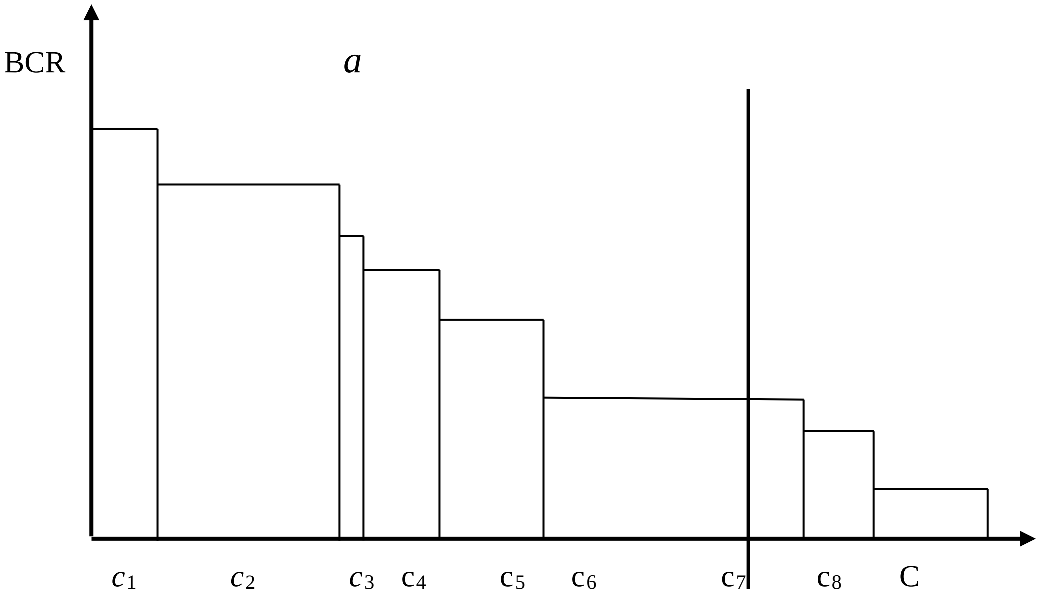

Diagram 1. 8 projects with costs along the C axis and benefit cost ratio along the BCR axis. Project 6 is only partially financed within the budget *a*.

2.2 The denominator

It is obviously of some importance to be clear about what budget constraint is considered as the binding one. It may be defined at the level of a plan or a programme or a whole sector of government. The higher the level, the more types of payment goes into the denominator ***c***. For instance, revenue from user charges must be included if they are earmarked to be used to finance the plan or to be used within the sector. If there is no earmarking, the revenues created by a project should not go into the denominator. They are however entered in the nominator, as part of the net benefits.

2.3 Maximising net present value of the plan within the budget constraint

We have just found that, given our assumptions, the net present value of a plan is maximised if projects are selected according to their BCR, defined as gros benefits divided by the discounted net payments that the agency must incur if this candidate project is included in the plan. It may be more intuitive to redefine $\boldsymbol{b}$ as the vector of net benefits, not gros benefits, and maximise net benefits instead.

To this end, define the net present value of project j as $b_j' = b_j - (1+\lambda)c_j$, where λ is the marginal cost of public funds, b_j is net benefits to travellers, transport operators and other affected parties, and c_j is the net present value of payments in and out over the relevant government budget. The net present value of the plan is

(2) $$NPV = \sum_{j=1}^{n} b_j' x_j = \sum_{j=1}^{n} \left(b_j - (1+\lambda)c_j\right)x_j$$

Our problem becomes

$$\text{(LP2)} \qquad \max_{\mathbf{x}} \ \sum_{j=1}^{n} b'_j x_j = \sum_{j=1}^{\mathrm{n}} \left(b_j - (1+\lambda) c_j \right) x_j \quad \text{s.t.} \ \sum_{j=1}^{n} \mathrm{c}_{\mathrm{j}} x_j \le a \ \text{ og } \ x_j \in [0,1] \ \ \forall j$$

Contrary to (LP1), some of the $b'_j x_j$ terms are likely to be negative, but these can be eliminated in advance without consequence for the optimal solution. The solution to (LP2) follows immediately from (LP1) by substituting b'_j for b_j. We have:

$$\text{(3)} \qquad \frac{b'_j}{c_j} = \frac{b_j - (1+\lambda) c_j}{c_j} = \frac{b_j}{c_j} - (1+\lambda)$$

The solution to (LP2) is therefore exactly the same as the solution to (LP1). All ratios are reduced by 1 + λ, but that does not affect the ranking. Thus if the BCR is properly defined and related to a single binding budget, as it should, there is no reason to make a distinction between the BCR and the criterion they call RNPSS (ratio of NPV to public sector support), as HEATCO (2006) does. Actually, official Norwegian guidance uses the (LP2) formulation instead of (LP1).[1]

What we have shown in this chapter, is that given the assumptions, any procedure that produce the solution (1) may be used, but no procedure that does not produce solution (1) is valid.

3 The case of mutually exclusive alternatives

According to HEATCO (2005a and b), quite a few countries point to the benefit-cost ratio as the only correct criterion to use if the objective is to maximise the net present value of a plan that is constrained by a single budget. Some, as an old Norwegian manual (Finansdepartementet 1979), even care to mention that this criterion breaks down if there are interdependencies between the projects. But none of them propose any alternative criterion for the case of mutually exclusive alternatives. This is our task in this section. We start by explaining the general idea, before formulating and solving the problem in a more formal way.

Let us assume that we have *n* – 1 independent candidate projects plus a candidate project number *n* with two mutually exclusive alternative designs. We order the *n* – 1 candidates by the BCR. We use the formulation of this criterion given in (3), so that zero is the demarcation point between profitable and unprofitable projects. Assume that the last

[1] See the website of the Norwegian Ministry of Finance. But the Ministry's experts were wrong in thinking that by substituting (LP2) solutions for (LP1) solutions, they had made a substantial improvement.

projects to fit into the budget all have BCR's close to a certain number *k*. Now we want to find out which one, if any, of the two alternative designs that deserve to be included in the plan at the expense of one or more of these marginal projects. If both alternatives have CBR below *k*, none of them qualifies. If only one of them has BCR above *k*, this alternative should be included at the expense of one or more of the projects whose BCR is *k*. What about the case where both alternatives have BCR's above *k*?

If we use a diagram similar to Diagram 1, with BCR on the vertical and *C* on the horizontal axis, the net present value of the whole plan is equal to the area of all colums to the left of the budget line *a*. It is this area that we want to make as large as possible. Let us say that one of the alternatives has a BCR considerably above *k*, while the other has has a somewhat lower BCR, but still above *k*. Obviously, we must choose the alternative with the largest area above the *k* level. The areas are not only dependent on the columns' heights (the BCR) but also on their widths (the C).

Diagram 2 shows a such case. The area DEFG in the diagram is a string og projects with a BCR of *k*. The cost of the projects with benefit D is c_0, and the costs of the projects with benefit E plus F is $c_2 - c_0$. The first competing alternative of project *n* has cost $c_1 - c_0$, a net present value of ABE and a BCR equal to the *height* of ABE. The second alternative has a cost of $c_2 - c_0$, a net present value of BCEF and a BCR equal to the height of CF (or BE).

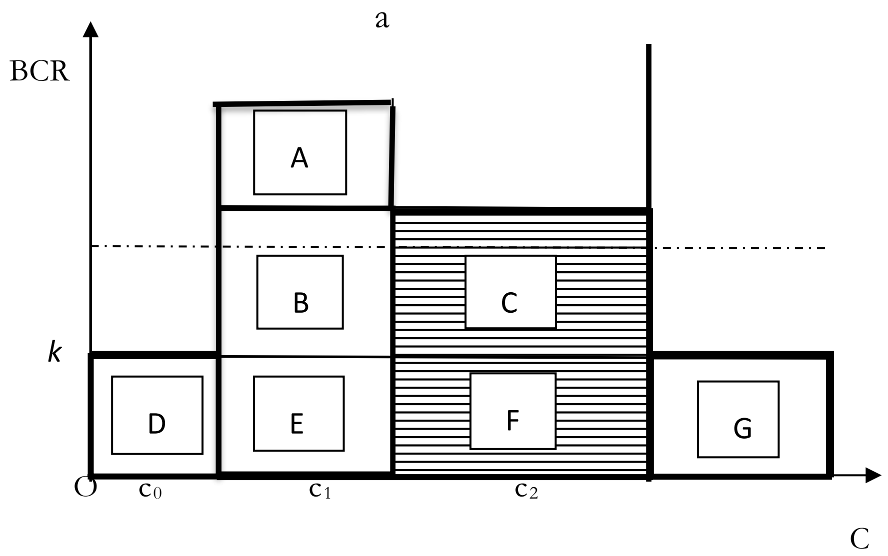

Diagram 2. A choice between mutually exclusive project alternatives ABE and BCFE.

As the area of A + B is smaller than the area of B + C, it is the second alternative that should be chosen, If however the *k* line had been at the dotted line instead, or even higher, the first alternative should be chosen. Thus the BCR of the marginal project included in the plan matters for the choice between the two alternatives of the n^{th} candidate project. The BCR of the two candidates is not a valid choice criterion in this case, or else project ABE would have won regardless of *k*. Likewise, the net present value is not a valid criterion, or else BCEF would have won in all cases. The reason is that the relative size of the boxes E and F has nothing to do with the choice here, since the net present value they represent is secured anyhow by choosing other projects than *n*. It is the area above the *k* level that matters.

Call the two alternatives *n1* and *n2*. The first alternative has a benefit above the *k* level (a surplus benefit) of W_{n1} = A + B, while the surplus benefit of the second alternative is W_{n2} = B + C. Let h_{in} denote the benefit cost ratio as defined by (3). We have:

$$W_{in} = \left(\frac{b_{in} - (1+\lambda)c_{in}}{c_{in}} - k\right)c_{in} = \left(\frac{b'_{in}}{c_{in}} - k\right)c_{ni} = \left(h_{in} - k\right)c_{ni},\ i = 1,2 \tag{4}$$

The indicator W_{in} can obviously be used to maximise economic efficiency when all projects except one are without alternatives. But we will now show that it is much more general than that. In fact, it is the choice criterion for all candidates to be included in a plan when at least one of them has mutually exclusive alternatives.

Assume that the plan in case is a national transport plan. Let *P* be the set of candidate projects, and index the elements of this set by $j \in P$. Call the set of mutually exclusive alternatives of project *j* by *Aj*, and index this set by $i \in Aj$. Observe that the number of elements in A_j may be different for each *j*, and that it may also consist of just one element. Let b_{ij} be the net present value of benefits to travellers, freight owners, transport sector operators and infrastructure providers from alternative *i* of candidate project *j*, and let c_{ij} be the net outlays over the constrained budget in case alternative *j* of project *i* is included in the plan.

We assume infinitely divisible projects and formulate the following linear programming problem to maximise the net present value of the plan:

$$\text{(LP3)} \qquad \max_{\mathbf{x}} \sum_{j \in P} \sum_{i \in A_j} \left[b_{ij} - (1+\lambda)c_{ij}\right]x_{ij} \ \text{ s.t. } \sum_{j \in P} \sum_{i \in A_j} c_{ij}x_{ij} \le a$$

$$\sum_{i \in A_j} x_{ij} \le 1, \quad j = 1,2,\ldots,|P|$$

$$x_{ij} \in [0,1] \quad \forall j \in P \text{ og } i \in A_j$$

Here, *a* is the budget and |*P*| is the number of candidate projects. Thus the first constraint is the budget constraint, while the second (or to be correct, the following |*P*| constraints) says the the fractions of each alternative design of a project must sum to at most 1. We will see that in practice, this implies that at most one alternative will be chosen.

To select just one alternative *i* for each of the *j* projects can sometimes be done in millions of ways. It is difficult to test out every possibility. Thus to solve the problem, we formulate a similar problem that under certain circumstances will produce the same solution as (IP1), but that is much easier to solve, at least approximately. What we do is to delete the budget constraint from the problem and include it in the objective function instead, multiplied by an unknown parameter that we shall call *k*. This procedure is called Lagrangian relaxation. The new objective function becomes:

$$V(k) = \sum_{j \in P} \sum_{i \in A_j} \left[b_{ij} - (1+\lambda) c_{ij} \right] x_{ij} - k \left(\sum_{j \in P} \sum_{i \in A_j} c_{ij} x_{ij} - a \right) \quad (5)$$

We may perform some simple rearrangements of the right hand side of (5):

$$V(k) = ka + \sum_{j\in P}\sum_{i\in A_j}\left[\left(b_{ij} - (1+\lambda)c_{ij}\right) - kc_{ij}\right]x_{ij} = ka + \sum_{j\in P}\sum_{i\in A_j}\left(\frac{b_{ij} - (1+\lambda)c_{ij}}{c_{ij}} - k\right)c_{ij}x_{ij}$$
$$= ka + \sum_{j\in P}\sum_{i\in A_j}\left(h_{ij} - k\right)c_{ij}x_{ij}$$

To optimise the modified objective function $V(k)$ for a given k means to find the optimal fractions of all the x_{ij}. Since k is a parameter and not a variable, ka is a constant that does not affect the optimisation. The sum of sums of the last line is easily seen as the sum over all projects and all project alternatives of the indicator W_{ij} of (4). Maximisation of this expression obviously means to select from each candidate project the alternative with the highest indicator value, then select projects according to the BCR, starting with the highest value and proceeding until the budget is exhausted. This procedure only requires identification, for each project, of the project alternative with the highest value, then a simple ordering of the projects. Obviously, it can be done by in an EXCEL spreadsheet.[2]

But this optimisation is conditional on k. Thus we must also choose the k that produces the best result. And for each chosen k, we need to repeat the same procedure to select the optimal set of x_{ij}'s. This choice of optimal k is also quite simple. The lowest possible value is $k = 0$. If the whole budget is not used up at this level of k, this is the optimal solution k^* (and the problem becomes just to find the alternative of each candidate project with the highest net present value, and add all such values that are above zero). If not, our first task is to find a k large enough that the budget is not used up. We have then an interval between 0 and this k on which the optimal k, k^*, must lie. On this interval, a search algorithm may then be applied to find the lowest k that does not use up the whole budget. The choices that follow from using this value of k, produces the optimal plan.[3]

Observe that for each new choice of k, the computation of all w_{ij} must be repeated. Then the sum of all cost for the projects with positive w_{ij} must be computed and compared to the given budget. The k is then adjusted so as to utilise as much as possible of the budget, but not more. This approach utilises the fact that the optimal k is both the Lagrangian multiplier of the budget constraint and the BCR of the last project to fit into the budget. A possible algorithm may consist of the following steps:

[2] The h_{ij}'s and the c_{ij}'s are output from the cost benefit analysis of every alternative of every candidate project, and is treated as given in the subsequent choice of projects and project designs for the plan.

[3] If projects are indivisible, our procedure does not garantee an optimal result, only a result close to the optimal. Since a small part of the budget is not used, it might be that some project that uses up more of the budget, but has a slightly lower indicator value than the ones we selected, might improve the economic value of the plan.

Sub routine

For any given k, (a) compute $w_{ij}(k)$ for all projects with project alternatives, (b) for all projects j, select the i with the highest $w_{ij}(k)$, (c) eliminate all remaining j with $w_{ij}(k) < 0$, and (d) add together all remaining c_{ij}. Call this sum $C(j;k)$.

Main routine

1. Set $k_0 = 0$. If $\sum_j C(j;k_0) < a$, compute the net present value of the optimal plan consisting of all selected ***ij*** with $w_{ij}(k_0) > 0$, and stop.
2. If $\sum_j C(j;k_0) > a$, make a guess at a $k_1 > k_0$. Perform the subroutine. If $\sum_j C(j;k_1) < a$ increase k_1 until $\sum_j C(j;k_1) > a$. Call this value of k_1 $\bar{k}$.
3. Use some search algorithm on the interval $[0,\bar{k}]$ to find points k_2, k_3 etc. with smaller and smaller $\left|\sum_j C(j;k_n) - a\right|$ until for k^* and some prespesified ε, $\varepsilon > a - \sum_j C(j;k^*) > 0$. Perform the subroutine and stop.
4. Compute the net present value of the optimal plan, $V(k^*)$.

This procedure secures that all eliminated projects with only one alternative have BCR < k^* and all eliminated alternatives are dominated by another alternative, while all retained projects and alternatives have $BCR \geq k^*$. Thus there is no point in substituting a fraction of an eliminated alternative for a fraction of some retained project or alternative. Furthermore, there is very little space between ***a*** and $\sum_j C(j;k^*)$, so it is not much point in squeezing in a fraction of a deleted project there. Therefore, the routine produces a plan that is as close to optimum as one wishes.

4 An example

The E39 is a major road in the western part of Norway. At present, a ferry carries the traffic on E39 across one of the major fjords in the area, Bjørnafjorden. The main issue in this study is to find the best conceptual solution for the crossing of Bjørnafjorden in the future. A cost benefit analysis is part of the quality assessment of this choice of concept.

The competing concepts studied are:

K2	minor improvements on the current road
K3	bridges from island to island in the outer part of the fjord

K4A and K4C long/short variants of a large bridge further into the fjord

K4D like K4C, except with ferry connection instead of bridge

K5A and K5B bridge solutions even further into the fjord

Table 1, based on Table 7-2 in Dovre and TØI (2012), shows the main results of the cost benefit analysis of different conceptual solutions in the E39 Aksdal-Bergen project.

From these numbers, we can compute h_i and c_i for i = 2, 3, 4A, 4C, 4D, 5A and 5B. We can then compute $w_i(k)$ for different values of k between 0 and 2. This is done in Table 2. To be precise: Investments and running costs from Table 1 are added to form c in Table 2. BCR in Table 1 is entered as h in Table 2. Finally, the indicator $w = (h - k)c$ is computed for the different values of k.

Table 1: E39 Aksdal-Bergen. Key numbers from the cost benefit analysis of the quality appraisal. (NOK Billion in 2012 prices).

	K2	K3	K4A	K4C	K4D	K5A	K5B
Investment	4,3	28,9	12,6	27,1	12,2	25,8	21,2
Running cost*	1,0	3,6	1,2	2,0	1,4	2,2	2,2
Gros benefit	5,5	62,9	26,7	56,5	34,8	55,7	52,3
Net present value	0,2	30,3	12,8	27,5	21,2	27,7	29,0
BCR**	0	1,0	1,0	1,0	1,7	1,1	1,4

* Maintenance and upkeep

** Benefit cost ratio (Net present value per NOK of National Road Authority budget outlays)

Table 2: E39 Aksdal-Bergen. Cost c (NOK billion in 2012 prices), Benefit cost ratio h and the indicator w(k) for the main alternatives at different values of k, the benefit cost ratio of marginal projects in the plan.

	K2	K3	K4A	K4C	K4D	K5A	K5B
c	5,3	32,5	13,8	29,1	13,6	28,0	23,4
h (=NNB)	0	1,0	1,0	1,0	1,7	1,1	1,7
w(0)	0	33	14	29	23	31	33
w(0,25)	-1	24	10	22	20	24	27
w(0,5)	-3	16	7	15	16	17	21
w(0,75)	-4	8	3	7	13	10	15
w(1)	-5	0	0	0	10	3	9
w(1,5)	-8	-16	-7	-15	3	-11	-2
w(2)	-11	-33	-14	-29	-4	-25	-14

We note in Table 2 that if the budget does not require projects to be more than just socially efficient (k = 0), there is a tie between K3 and K5B. If we strengthen our reqirement one notch, our rule will pick K5B alone as the best alternative solution. As k approaches 1, K4D – the alternative with the highest benefit cost ratio – will take over the lead. This is all as expected.

Actually, the alternative that was chosen was K4C. As we can see from Table 1, it has neither the highest net present value nor the highest benefit cost ratio. Alternative K3 was however eliminated because of unacceptable non-monetarised effects. This having been done, K4C emerged as one of several alternatives with about the same net present value. It was also this alternative that answered best to the purpose of the project, which was to build a fast connection between Stavanger and Bergen, the two major cities on the west coast. If this is the purpose of the project, there is of course nothing to prevent it from becoming the the decisive factor in the end. It will always be necessary to use judgement to supplement formal methods. But the reasons for the final choice should of course always be stated clearly.

K4D is identical to K4C except that it retains the ferry crossing. Thus it can function as a first stage in the construction of K4C, postponing the bridge until traffic levels have grown sufficiently. We see that as the budget gets tighter and only extremely profitable projects can be realised, it is this first stage that becomes the best alternative. If we only compare K4C and K4D, K4C should be chosen for $0 \le k < 0,5$, while K4D takes over when $k \ge 0,5$. Thus, the tightness of the budget and the amount of profitable projects elsewhere have a bearing on the question of whether we should opt for a simple and cheap or a more expensive but better solution. This perspective has not been used explicitly on the choice of alternative in any project up until now, as far as I know.

At *k* near zero, our criterion becomes similar to the net present value criterion, while as *k* increases, it becomes more like the benefit cost ratio, Thus the global setting into which the project competes for funding, matters for the criterion to be used locally.

5 Remarks

5.1 Cases with mutually exclusive alternatives

We assumed that the task at hand was to select projects to the national transport plan. But there are many similar situations. In the initial exploratory planning stage, for instance, there are always competing designs of the project, and very often, it is clear that there exists at least an expectation that the amount of funds to be spent on transport investments is kept within certain limits. An urban transport plan is a case in point. In urban areas, there are also often interdependencies between projects either on the demand side or in construction. If very many projects depend on each other, one should formulate and solve a network design problem. If however the interdependencies consist of a number of small groups of interdependent projects, another possibility is to construct all possible combinations of the projects within a group. These form mutually excluding alternatives that should compete with the single projects and the combinations of other groups as outlined in (LP3).

5.2 Several constraints

In addition to the budget constraint, there may be targets in the form of constraints on for instance CO_2 emissions, the number of accidents etc. In that case, neither the BCR nor the indicator of section 3 is of any use. If there are just two constraints, we could probably add them both to the objective function, each with its own Lagrangian parameter. But the search procedure would be more difficult. Anyhow, a linear programming problem can always be formulated and solved with any commercial software for LP problems.

5.3 Who is responsible for the selection?

It is worth noting that unless we can propose a correct value of k^* in advance, the procedure of section 3 implies that, except for the projects with only one alternative, the final selection of the right alternative design can no longer be taken locally, but must be transferred to a central authority.

6 Conclusion

If the projects are infinitely divisibe and independent and the given budget is the only binding restriction, and if none of the projects exist in mutually exclusive alternatives, maximum total net present value of the plan is achieved by ranking them according to their benefit cost ratio and selecting them from the top until the budget is exhausted. If there are mutually exclusive alternatives, neither the net present value nor the cost benefit ratio is a valid selection criterion. Instead, a selection criterion (a special indicator) that depends on the benefit cost ratio of the last project that is included in the plan should be applied.

Initially, assume this marginal BCR to be given. Then for each project, the alternative with the highest score on the selection criterion should be chosen. When this has been done, compute the sum of costs of all projects with an indicator value above zero, or alternatively with a BCR above the assumed marginal BCR. Adjust the assumed marginal BCR up or down and repeat the computation until until the cost of all projects with a positive indicator value is just a little below the given budget. This procedure utilises the fact that, by construction, the Lagrangian multiplier of the budget constraint and the BCR of the last project to fit into the budget are the same.

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