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Shadow pricing of electricity generation using stochastic and deterministic materials balance models

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Highlights

- Shadow pricing is a popular method for abatement cost estimation.
- Most previous studies ignore the materials balance condition.
- Materials balance consistent Convex Nonparametric Least Squares estimator developed.
- Shadow prices for 160 US power producers from stochastic and deterministic models.
- Abatement costs severely understated when the materials balance is ignored.

Abstract

Marginal abatement cost is an essential input to optimal environmental policies. Shadow pricing has become a popular method for estimating abatement costs subject to parsimonious data requirement. This paper provides a novel contribution to the literature on shadow pricing by considering the implication of the materials balance principle for shadow prices. To that end, the paper establishes a Convex Nonparametric Least Squares estimator for the weak G-disposable production model, which for the first time enables modeling a composite error term and joint estimation of the production frontier and contextual variables within this production model framework. Applying the Directional Distance Function, environmental efficiencies and shadow prices for carbon dioxide emissions are estimated for a sample of power producers using both stochastic and deterministic frontier models. Average shadow price estimates for carbon dioxide range between 14,000 and 40,000 \$/ton CO_2 for the weak G-disposable model and between 70 and 77 \$/ton CO_2 for the conventional production model that ignores the materials balance. These findings cast doubt on previous shadow price estimates since a majority of comparable studies ignore the strict technical relationship among pollution-generating inputs and bad outputs under the materials balance condition.

Keywords

Data Envelopment Analysis; Convex Nonparametric Least Squares; Weak G-disposability; Shadow Price; CO_2

JEL

codes: D24; C61; Q53

Nomenclature

Glossary

Abbreviation

Term

CNLS
: Convex Nonparametric Least Squares

CO_2
: Carbon dioxide

DEA
: Data Envelopment Analysis

DGP
: Data generating process

DMU
: Decision making unit

FD
: Free disposability

MBC
: Materials balance condition

SFA
: Stochastic Frontier Analysis

StoNED
: Stochastic Nonparametric Envelopment of Data

WGD
: Weak G-disposability

1. Introduction

There are several approaches to incorporate bad outputs in production analysis; see Dakpo et al. [1] for an overview. In recent years, their consistency with the *materials balance principle* has received attention. Coelli et al. [2] paved the way for a new class of environmental production models based on the materials balance. Rødseth [3] analyzed a set of axioms, broadly referred to as the *weak G-disposable* production model, and showed that they are in compliance with the materials balance principle. Hampf and Rødseth [4] showed how Rødseth's theoretical model could be implemented empirically using Data Envelopment Analysis (DEA). While there have been several empirical studies following Hampf and Rødseth's paper, DEA continues to be the workhorse of applied production analysis using weak G-disposability. A noteworthy exception is the recent study by Atkinson and Tsionas [5] that embeds the materials balance principle into a Stochastic Frontier Analysis (SFA) framework. Their approach is, however, not readily comparable to Hampf and Rødseth's (2015) DEA model.

DEA has its roots in the seminal work of Farrell [6], but was coined by Charnes et al. [7]. A major advantage of this approach is that it is nonparametric, thereby avoiding assumptions about the functional form of the production model. On the other hand, it does not accommodate stochastic noise in the data – i.e., it is a *deterministic* model – but regards any deviation from the frontier as inefficiency. This is a potential drawback in the current context as parameters of the materials balance or materials balance-based estimates of bad outputs are likely vulnerable to some level of measurement bias. Another drawback is that the DEA literature predominately models the impacts of contextual variables on efficiencies using a two-stage approach [cf. 8]. Johnson and Kuosmanen [9] find that unbiased and efficient estimation of the impact of contextual variables on efficiency requires joint estimation of the production frontier and effects of contextual variables. The use of a one-stage approach for modeling contextual variables within the weak G-disposability framework is discussed by Hampf and Rødseth [10], who note.

"*A one-stage analysis could have been conducted by including the second-stage variables in Convex Nonparametric Least Squares analysis (*i.e.*, the StoNED model by Kuosmanen and Kortelainen* [11]*) as proposed in Johnson and Kuosmanen* [12]*. Moreover, this approach would have enabled both deterministic and stochastic specifications of the technology*." [10, p. 623–624].

However, the main restriction is that:

"*the MBC-model*[1] *is currently not developed for StoNED*" [10, p. 624].

The current paper provides a cure for this problem. It starts by deriving the dual of Hampf and Rødseth's [4] DEA model. Drawing on Kuosmanen [13] and Kuosmanen and Johnson [14], a Convex Nonparametric Least Squares (CNLS) estimator for the weak G-disposable model is then derived. Thereby, this paper enables efficiency analysis accommodating both stochastic modeling and one-stage estimation of the impact of contextual factors while ensuring compliance with the materials balance principle.

Marginal abatement cost refers to the (minimum) cost of a unit reduction of a bad output, which is an essential input to environmental policy. While there are multiple methods available for estimating abatement costs, they can broadly be classified into economics and engineering approaches [15]. Methods range from techno-economic evaluation (see e.g. Dai et al. [16] for a recent example) and programming to data-driven methods such as econometrics and production analysis [17]. In this study, we concentrate on production analysis, which has become a popular data-driven approach for estimating marginal abatement costs by means of *shadow pricing*. Recent publications on shadow pricing of carbon dioxide include Ao et al., [18], Yue et al. [19], and Zhao and Qiao [20], just to mention a few among a substantial number of recent studies.

The standard definition of marginal abatement costs in the shadow pricing literature concerns forgone revenue from reducing emissions by one unit [21]. This notion is also maintained in the recent study by Shen et al. [22] that explores shadow pricing in the context of the by-production approach to environmental efficiency analysis [23]. In contrast, Kuosmanen and Zhou [17] recently advance the shadow pricing approach by i) considering multiple abatement strategies and ii) identifying shadow prices at local (i.e., quantile) frontiers.

In line with most previous studies (see Zhou et al. [24] for a review) Kuosmanen and Zhou [17] model inputs (as well as pollutants) as freely disposable. They argue there is a trade-off between letting the data speak for themselves and force the data to a straitjacket of too tight theoretical equations. However, assuming free disposability ignores the tight technical relationship among pollution-generating inputs and bad outputs – governed by the materials balance principle – as one variable can vary while the others are maintained at current levels.

A second objective of this paper is consequently to examine the profound implications of the dependence of bad outputs (e.g., carbon dioxide) on pollution-generating inputs (e.g., coal) for shadow pricing. Simply put, the standard notion in the shadow pricing literature – that bads can be reduced by lowering good outputs for given inputs – is substantially handicapped when the amount of bads produced is close to predetermined by the input mix. This assumption contradicts the materials balance principle, which enforces limited substitutability among material inputs and bads and consequently precludes that bads can be reduced for given inputs. Its consequences are illustrated by comparing shadow prices for carbon dioxide emissions estimated assuming weak g- and free disposability using stochastic and deterministic CNLS production models applied to Hampf's and Rødseth's [4] dataset containing 160 US bituminous-fired electricity generating units. Shadow price estimates are substantially higher when the technical relationship among pollution-generating inputs and bads is explicitly modeled, which casts doubt on previous comparable shadow pricing estimates presented in the literature.

A main contribution of this paper is to derive a CNLS estimator for the weak G-disposable production model, which for the first time enables modeling a composite error term and joint estimation of the production frontier and the impact of contextual variables within this production model framework. Second, the paper offers a first investigation into the consequences of the materials balance principle for shadow pricing of bad outputs. Thereby, the paper provides novel insights for and puts a caveat on previous studies in the field of shadow pricing, which is a substantial and rapidly growing field within abatement cost analysis.

This paper is structured as follows. Section 2 presents the theoretical underpinnings of the shadow pricing analysis and develops the CNLS estimator for weak G-disposability. Section 3 presents the dataset and the results of a Monte Carlo study that sheds light on differences among how the deterministic materials balance and conventional production models characterize substitutability among intended and unintended outputs and their implications for efficiency measurement, while Section 4 reviews the empirical results. Section 5 offers conclusions and recommendations for further research.

2. Theoretical underpinnings

We tailor the theoretical production model to the empirical case study, i.e., electricity production. The selected input and output variables are in line with previous studies on US power generation [25]. Generalization of the model and the theoretical results to a higher number of inputs and outputs is straightforward.

2.1. Production theory

Let $\boldsymbol{x} \in \mathfrak{R}_+^2$ denote a vector of inputs, of which $\boldsymbol{x_P} \in \mathfrak{R}_+$ is a material input (i.e., fuel) and $\boldsymbol{x_{NP}} \in \mathfrak{R}_+$ is a non-material input (i.e., generating capacity). We will also refer to the former as the pollution-generating input. Let $\boldsymbol{y} \in \mathfrak{R}_+$ denote the desirable output (i.e., electricity), and let the sole pollutant analyzed (i.e., carbon dioxide, abbreviated CO_2) be denoted $\boldsymbol{b} \in \mathfrak{R}_+$. A technology set that summarizes technically feasible input–output combinations is defined by Eq. *(1)*.

$$T = \{(\boldsymbol{x}, y, b) \text{: } \boldsymbol{x} \text{ can produce } (y, b)\} \tag{1}$$

The technology is assumed to possess a set of properties, known as axioms. We refer to Färe and Primont [26] for details about standard axioms in production theory and Rødseth [3] for their compliance with the materials balance principle. In this paper, we study and compare production possibilities under two different sets of axioms (i.e., for two distinct technologies). In both cases, the technology is assumed to exhibit convexity and free disposability of the desirable output. Following Kuosmanen and Zhou [17], the first technology specification – the free disposability (FD) model – treats inputs and the bad output as freely disposable. That is,

$$\text{if } (\boldsymbol{x}, y, b) \in T \text{ and } \boldsymbol{x}' \geq \boldsymbol{x}, b' \geq \text{b}, \text{ then } (\boldsymbol{x}', y, b') \in T \tag{2}$$

In words, if the vectors indicated by primes contain elements of equal or larger quantities compared to the vectors not indicated by primes, the vectors indicated by primes are included in the technology. Note that free disposability does not preclude increasing the consumption of the material input without affecting the production of the bad output, and vice versa. In this sense, the two variables are “technically decoupled” in the realm of the free disposable model.

The second model considered is Rødseth’s [3] weak G-disposability (WGD) framework. To define the weak G-disposable axiom tailored to our use case we need to introduce additional notation. Let $\boldsymbol{u} \in \mathfrak{R}_+$ denote the material flow coefficient for the material input, describing the carbon content and therefore CO_2 emissions per unit of fuel. The weak G-disposability axiom can consequently be defined:

$$\begin{aligned}&\text{if } (\boldsymbol{x}, y, b) \in T \text{ and } x_P' \geq x_P, b' \geq \text{b}, \text{u}x_P' - b' = 0,\\ &\text{then } (\boldsymbol{x}', y, b') \in T\end{aligned} \tag{3}$$

Weak G-disposability is a *directional disposability* condition that in this case enforces disposability according to the materials balance principle, i.e., ensuring that carbon entering the system through fuel consumption is unavoidably converted into CO_2. Hence, all points included in the technology set are subject to a strict technical relationship among material inputs and pollution.

The aim of the subsequent empirical analysis is to illustrate how the technology and its characterizations of output substitutability vary when evoking the fundamentally different disposability assumptions represented by Eqs. (2)-(3). In the former case, there is no firm relationship among material inputs and pollution, while the latter case imposes a strict correspondence between use of material inputs and pollution for any input–output combination. By comparing the two production models with regards to output substitutability, this study expands the scope of Atkinson’s and Tsionas’s [5] recent research that focused on the impact of incorporating the materials balance on productivity and efficiency.

While the technology set is a theoretical construct, a function representation that can be estimated from data is required for empirical analysis. We use a Directional Distance Function [27] that nowadays has become a standard tool for shadow pricing [24]. Following the standard convention [e.g., 21], our primary model considers a Directional Distance Function that expands the desirable output and contracts the undesirable output to the production frontier according to the direction vector $(\mathbf{g}_x, g_y, g_b) = (\mathbf{0}, 1, 1)$. In this case, the Directional Distance Function is defined:

$$\vec{D}(\boldsymbol{x}, y, b; \mathbf{0}, 1, 1) = \sup\{\theta : (\boldsymbol{x}, y + \theta, b - \theta) \in T\} \tag{4}$$

where $\theta \in \mathfrak{R}$ measures the maximal expansion of the desirable output and contraction of the bad output along the selected path (or direction vector) to the production frontier. The distance function inherits the properties of the technology set and is greater or equal to 0 when $(\mathbf{x}, y, b)$ is an element of the technology. $\theta = 0$ indicates that the unit under observation operates on the frontier, while a positive value signals inefficiency in production. While we apply the most common direction vector for our main analysis, other direction vectors are subsequently considered as robustness checks.

2.2. Shadow pricing

Following the influential publication by Färe et al. [21], we derive shadow prices using duality theory. Since the Directional Distance Function completely characterizes the technology set, the revenue function can be defined

$$R(\boldsymbol{x},p,q)=\sup_{y,b}\left\{py-qb:\vec{D}(\boldsymbol{x},y,b;\mathbf{0},1,1)\geqslant 0\right\} \tag{5}$$

where $p\in\mathfrak{R}_{+}$ and $q\in\mathfrak{R}_{+}$ are (shadow) prices for the good and bad outputs, respectively. Utilizing first order conditions, the shadow price for the bad output is defined [cf., 21]:

$$q=-p\left[\frac{\partial\vec{D}(\boldsymbol{x},y,b;\mathbf{0},1,1)/\partial b}{\partial\vec{D}(\boldsymbol{x},y,b;\mathbf{0},1,1)/\partial y}\right] \tag{6}$$

The shadow price defined by Eq. (6) represents the value of the intended output that must be forgone to achieve a marginal reduction of the bad output for given inputs.

As noted by Kuosmanen and Zhou [17], similar shadow pricing rules can also be defined based on the substitutability among inputs and the bad output. They argue that the minimum of the set of admissible shadow prices should be regarded marginal abatement costs. We agree with their opinion, but to relate our main result to the approach used by most existing studies we follow the standard convention and base shadow prices for bads solely on the trade-off among the good and bad outputs.

Our emphasis is on how weak G-disposability affects the shadow price for the bad output according to Eq. (6), relative to the standard case of imposing free disposability of material input. To the best of our knowledge, there is no comparable empirical study that estimates shadow prices subject to the materials balance condition. Rødseth [28] derives shadow pricing rules in the context of the materials balance condition, but his contribution is purely theoretical.

2.3. Nonparametric estimation

In this section, we first derive the dual of the DEA estimator of the Directional Distance Function under weak G-disposability. Based thereon, we develop a CNLS estimator for the weak G-disposable model.

2.3.1. Weak G-disposable DEA model

Assume a dataset consisting of l=1,..,L decision making units or DMUs for short. Following Hampf and Rødseth [4], the weak G-disposable DEA estimator of the Directional Distance Function for DMU l' under variable returns to scale corresponds to solving the following linear programming problem:

$$\vec{D}\left(\boldsymbol{x}^{l'},y^{l'},b^{l'};\mathbf{0},1,1\right)=\max_{\boldsymbol{\varepsilon},\theta,\boldsymbol{\lambda}}\{\theta: \begin{array}{l}\sum_{l=1}^{L}\lambda^l y^l\geq y^{l'}+\theta\\ \sum_{l=1}^{L}\lambda^l b^l+\varepsilon_b=b^{l'}-\theta\\ \sum_{l=1}^{L}\lambda^l x_P^l+\varepsilon_P=x_P^{l'}\\ \sum_{l=1}^{L}\lambda^l x_{NP}^l\leq x_{NP}^{l'}\\ \sum_{l=1}^{L}\lambda^l=1\\ u\varepsilon_P-\varepsilon_b=0\\ \lambda\geq 0,\varepsilon_b\geq 0,\varepsilon_P\geq 0\}\end{array} \tag{7}$$

In this formulation, $\lambda^l,\forall l$ denote intensity variables that enable linear combinations of datapoints to form the production frontier, while $(\varepsilon_p,\varepsilon_b)$ represent slacks in production. After replacing ε_b by $u\varepsilon_P$ in the constraint for the bad output and removing $u\varepsilon_P-\varepsilon_b=0$ from Eq. (7) for simplicity, the dual of this linear program can be written

$$\vec{D}\left(\boldsymbol{x}^{l'},y^{l'},b^{l'};\mathbf{0},1,1\right)=\begin{array}{l}\min_{\gamma,\delta,\alpha,\nu}\{\alpha+\gamma_P x_P^{l'}+\gamma_{NP}x_{NP}^{l'}+\nu b^{l'}-\delta y^{l'}\\ s.t.\\ \alpha+\gamma_P x_P^l+\gamma_{NP}x_{NP}^l+\nu b_l-\delta y_l\geq 0,l=1,..,L\\ \gamma_P+\nu u\geq 0\\ v+\delta=1\\ \gamma_{NP}\geq 0,\delta\geq 0\}\end{array} \tag{8}$$

where $(\alpha,\gamma_P,\gamma_{NP},\nu,\delta)$ are dual variables or prices, also known as multipliers or weights.

$\gamma_P+\nu u\geqslant 0$ imposes weak G-disposability by means of restricting the relationship among dual prices of material input and pollution, depending on the material flow coefficient, u. Omitting this inequality enables modeling the DEA technology under freely disposable material input and pollution. In the case of free disposability, dual prices for material input and pollution are constrained to be non-negative.

2.3.2. CNLS models

The DMU-specific DEA model can be estimated jointly for all L DMUs by formulating the corresponding CNLS quadratic optimization problem.

Proposition: The minimization problem in Eq. (8) can be equivalently written as a CNLS model with a one-sided error term. Specifically, the DEA efficiency scores can be obtained as the optimal solution to Eq. (9).

$$\begin{aligned}
&\min_{\gamma,\delta,\alpha,\nu,\kappa} \sum_{l=1}^{L} \kappa^{l^2} \\
&s.t. \\
&\delta^l y^l = \alpha^l + \gamma_P^l x_P^l + \gamma_{NP}^l x_{NP}^l + \nu^l b^l - \kappa^l, \forall l \\
&\alpha^l + \gamma_P^l x_P^l + \gamma_{NP}^l x_{NP}^l + \nu^l b^l - \delta^l y^l \le \alpha^{l'} \\
&\quad + \gamma_P^{l'} x_P^l + \gamma_{NP}^{l'} x_{NP}^l + \nu^{l'} b^l - \delta^{l'} y^l, \forall l, l' \\
&\gamma_P^l + v^l u \ge 0, \forall l \\
&v^l + \delta^l = 1, \forall l \\
&\gamma_{NP}^l \ge 0, \delta^l \ge 0, \kappa^l \ge 0, \forall l
\end{aligned} \tag{9}$$

Proof: See Kuosmanen [13], Appendix 1.

Note that $\kappa^l, \forall l$ can be interpreted as a residual in a regression analysis, where the objective is to minimize the sum of squared errors. However, because the error terms are restricted to be non-negative, i.e., $\kappa^l \geqslant 0, \forall l$, they correspond to DEA estimates of the Directional Distance Function. Hence, the error terms take the value 0 for efficient units and are >0 for inefficient units.

Following Kuosmanen and Zhou [17], the corresponding CNLS problem for the free disposable technology is defined:

$$\begin{aligned}
&\min_{\gamma,\delta,\alpha,\nu,\kappa} \sum_{l=1}^{L} \kappa^{l^2} \\
&s.t. \\
&\delta^l y^l = \alpha^l + \gamma_P^l x_P^l + \gamma_{NP}^l x_{NP}^l + \nu^l b^l - \kappa^l, \forall l \\
&\alpha^l + \gamma_P^l x_P^l + \gamma_{NP}^l x_{NP}^l + \nu^l b^l - \delta^l y^l \le \alpha^{l'} \\
&\quad + \gamma_P^{l'} x_P^l + \gamma_{NP}^{l'} x_{NP}^l + \nu^{l'} b^l - \delta^{l'} y^l, \forall l, l' \\
&\nu^l + \delta^l = 1, \forall l \\
&\gamma_{NP}^l \ge 0, \delta^l \ge 0, \kappa^l \ge 0, \forall l \\
&\gamma_P^l \ge 0, \nu^l \ge 0, \forall l
\end{aligned} \tag{10}$$

The estimated dual prices for the good and bad outputs from Eqs. (9)-(10) are used to calculate shadow price for the bad output according to Eq. (6).

2.3.3. Stochastic modeling and contextual variables

We consider two extensions of the optimization problems in Eqs. (9)-(10).

- **Two-sided error term**: Mean value estimators of the weak G- and free disposable technologies are obtained by omitting restrictions $\kappa^l \geqslant 0, \forall l$ from Eqs. (9)-(10). This enables estimating the weak G-disposability model assuming a composite error term, and ultimately to obtain efficiency scores in a second stage estimation [11].

- **Contextual variables**: By appending a linear function $g(z) = \sum_{k=1}^{K} \varpi_k z_k^l$ to the regression equations in (9)-(10), the effects of contextual variables can be estimated jointly with the residuals (i.e., efficiency scores); cf. Johnson and Kuosmanen [9]. Here, $\varpi_k, \forall k$ denote unknown parameters in a conventional regression analysis. Contextual factors are considered both for the stochastic and deterministic production models.

3. Case study and data

We base the empirical study on the dataset originally collected by Hampf and Rødseth [4] and extended with price data in Hampf and Rødseth [25]. These studies focus on economic consequences of ambitious new regulation for greenhouse gas emissions launched by the Obama administration, but which was later mitigated by the Trump administration. With seemingly negligible changes in central government policies to control carbon dioxide emissions from US power plants in recent years, we still find the dataset relevant for CO_2 abatement cost estimation.

The dataset comprises 160 US electricity generating units in operation in 2011. Key selection criteria for the sample are: Bituminous-fired coal generators only; Nameplate capacity larger or equal to 10MW; Pulverized coal-fired generating units only; Subcritical units only. These are implemented to ensure a more homogenous sample than what is normally used for efficiency analysis of power plants. We refer to Hampf and Rødseth [4] for further elaboration and for details regarding the selection of variables and collection and processing of data. Key data sources are forms EIA-860_Generator and EIA-861, as well as EPA's database "Air Markets Program Data".

Table 1 presents an overview of the variables used for the empirical analysis. As previously outlined, the empirical production technology comprises two inputs (capacity and fuel) that produces two outputs (electricity and CO_2). To consider vintage effects as well as the impacts of mitigation of other pollutants under regulation (i.e., sulfur dioxide, SO_2, and nitrogen oxides, NO_x), age and local emission intensities are modeled as contextual variables. Finally, electricity prices are used to derive shadow prices for CO_2, cf. Eq. (6). They are set equal to the average of retail and resale prices of electricity per generating unit; see Hampf and Rødseth [25] for details.

Table 1. Summary statistics.

Type	Variable	Unit	Obs	Mean	St.Dev	Min	Max
Input	Capacity	MW	160	337.9	231.8	100.0	1425.6
	Fuel	1,000 MMBtu	160	16,089.1	14,715.8	57.4	74,900.0
Output	Electricity	100 MWh	160	16,967.2	16,165.2	58.8	85,413.0
	CO_2	1,000 Tons	160	1,653.2	1,509.6	5.9	7,686.1
Contextual	Age	Years	160	47.3	10.8	18.0	64.0
	SO_2-ratio	1,000 tons per MWh	160	4.4	4.8	0.1	25.9
	NO_x-ratio	1,000 tons per MWh	160	1.4	0.8	0.2	4.0
Price	Electricty price	$ per MWh	144	68.4	10.0	42.1	99.7

3.1. Empirical relationship among coal consumption and CO_2

Before turning to the Directional Distance Function results we use the dataset to visualize the persistent technical relationship that exists among the fuel input and CO_2 emissions. This is due to CO_2 being a product of the carbon content of the fuel. Fig. 1 presents a scatterplot of the two variables for the 160 observations contained in the dataset. Regressing fuel consumption on CO_2 emissions provides a perfect fit (i.e., R-squared equal to 1), while the effect of fuel use on CO_2 emissions is found to be statistically significant at the 1-percent level by means of t-testing. We consequently consider weak G-disposability a highly relevant axiom for this case.

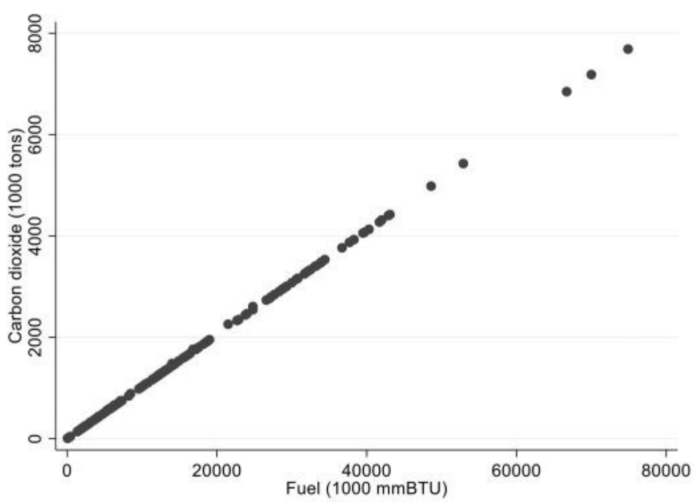

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Fig. 1. Correlation among fuel consumption and carbon dioxide emissions. Source: Own dataset.

The fuel's carbon content depends to some extent on the individual coal mine. Hampf and Rødseth [10] analyze the impact of fuel quality differences on environmental efficiency of CO_2 and SO_2 emissions from power plants. They conclude that while fuel quality differences matter for SO_2 efficiency it has a negligible impact on CO_2 efficiency. Based thereon, we do not pay attention to (negligible) fuel quality differences in this study. We therefore set the material flow coefficient for the fuel input equal to the sample average CO_2-to-fuel ratio.

In the long run, end-of-pipe abatement technologies for CO_2 will enable breaking the technical dependance of CO_2 on fuel consumption. As they still are not widespread – and based on Fig. 1 – it is reasonable to assume that a linear technical relationship among fuel and CO_2 exists for the generating units in the sample. This is also the case for previous comparable studies focusing on shadow pricing of CO_2. Weak G-disposability preserves this relationship for any feasible production plan.

3.2. Monte Carlo simulation

To further illustrate how the technical relationship among coal consumption and CO_2 emissions are treated differently by the WGD and FD models, we undertake a small Monte Carlo simulation study in which the Data generating Process (DGP) maintains the relationship among coal consumption and CO_2 presented in Section 3.1. The DGP considered is inspired by the Monte Carlo study by Hampf [29], and can be considered a special case of the by-production approach to modeling of bad outputs [23]. This approach considers separate production relations for good and bad outputs, respectively.

$$\begin{aligned} y &= x_P^{0.79} x_{NP}^{0.19} e^{-\kappa_y} \\ b &= 0.1028992 x_P + \kappa_b \end{aligned} \tag{11}$$

where κ_y and κ_b refer to inefficiencies in the generation of the intended and unintended outputs, respectively.

The DGP under consideration is presented by Eq. (11), which contains parameter values that are tailored to the data presented in Table 1. The Cobb-Douglas production function for the good output is fitted using Ordinary Least Squares, which gives an adjusted R^2 of 0.99. The emission coefficient is set equal to the sample average (i.e., 0.1028992). Deviations from mean emission factor and Cobb Douglas function (i.e., inefficiencies), alongside observed min and max values for the two inputs, are used to determine admissible ranges of inputs (x_P, x_{NP}) and efficiency parameters (κ_y, κ_b). These are all drawn from the uniform distribution over the intervals [57,74900] (energy input), [100,1425] (capacity input), [0,1.3] (good output efficiency), and [0,0.005] (bad output efficiency).

Note that the bad output efficiency parameter κ_b allows minor variation in the emission coefficient of the unit under consideration. However, it does in general assume away inefficiency in pollution generation. That is, burning a ton of coal leads to a close to predetermined amount of emissions. Consequently, the only way to reduce emissions is through reducing coal consumption (in absence of end-of-pipe abatement).

In turn, this means that the potential for increasing the good output by the same amount as the reduction in pollution – as considered by the Directional Distance Function defined by Eq. (4) – is severely limited. To calculate the "true" Directional Distance Function, we first calculate deviations per output based on the simulated data. We use tilde to indicate simulated data, and define

$$\begin{aligned} \theta_y &= \tilde{x}_P^{0.79} \tilde{x}_{NP}^{0.19} - \tilde{y} \\ \theta_b &= \tilde{b} - 0.1028992 \tilde{x}_P \end{aligned} \tag{12}$$

and subsequently identify the minimal distance as the true Directional Distance Function, i.e., $\theta = \min\{\theta_y, \theta_b\}$.

The DGP in Eq. (11) assumes away random noise, and we consequently use deterministic models defined by Eqs. (9)-(10) to a) estimate Directional Distance Functions for the WGD and FD models based on synthetic data and b) to evaluate Mean Squared Errors of the WGD and FD estimators. We simulate data for three different sample sizes (L=25; 50; 100) with 500 replications for each sample size. Mean Squared Errors for each of the simulations are presented by Table 2, which shows that the WGD model has a negligible error due to ensuring the technical relationship among coal consumption and CO_2 emissions. The FD estimator, on the other hand, performs poorly, and its Mean Squared Error increases with sample size.

Table 2. Results of Monte Carlo simulations with 500 replications (Mean Squared Errors).

Model	***L*=25**	***L*=50**	***L*=100**
WGD	0.000	0.000	0.000
FD	1,973,286.651	2,471,665.329	2,845,920.343

4. Empirical results

Eqs. (9)-(10) are fitted using GAMS, considering the following model specifications:

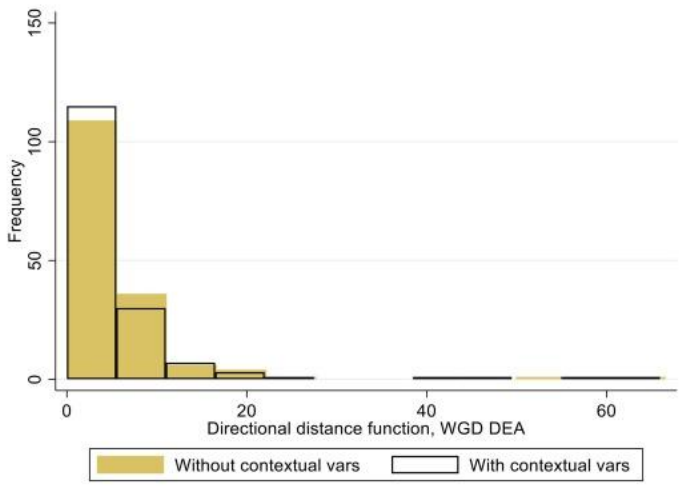
- Weak G-disposability (WGD) and free disposability (FD)
- One-sided errors (i.e., deterministic DEA) and two-sided errors (i.e., stochastic CNLS)
- With and without contextual variables

Consequently, 8 different production models are fitted under the assumption of variable returns to scale. The code used for estimation can be found in Appendix A.

4.1. Efficiencies

First, we compare DEA estimates of the Directional Distance Function – i.e., the one-sided errors from Eqs. (9)-(10) – calculated under various scenarios. We refer to Eq. (4) for a formal definition of the Directional Distance Function, which is our preferred efficiency measure. As previously described, it is used for evaluating the technical potential for jointly expanding the desirable output and contracting the undesirable output according to the direction vector $(\mathbf{g}_x, g_y, g_b) = (\mathbf{0}, 1, 1)$. The efficiency measure is greater or equal to 0, where 0 indicates efficient production.

Fig. 2 shows WGD efficiencies with and without joint estimation of effects of contextual variables. The Directional Distance Function estimates range between 0 and 67, with most of the units being very efficient. While modeling contextual variables slightly improves efficiencies, the overall efficiency distribution is affected only to a minor degree. Hence, estimating the models with and without contextual variables matters very little for the efficiency analysis.

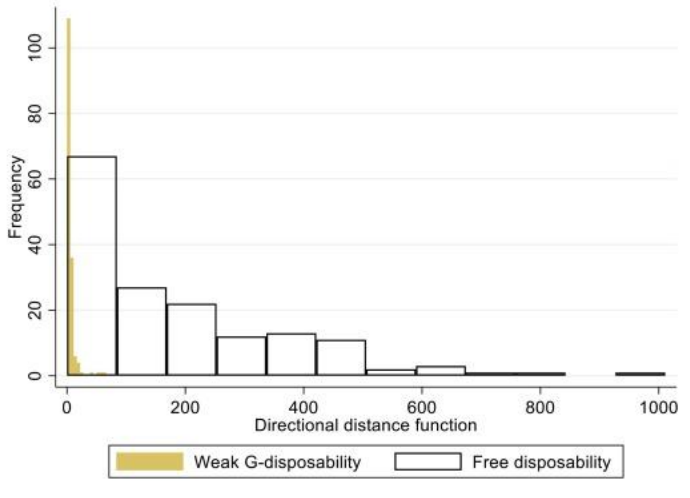


Download : Download high-res image (123KB)

Download : Download full-size image

Fig. 2. Distribution of WGD DEA efficiencies with and without simultaneous estimation of effects of contextual variables.

Fig. 3 compares environmental efficiencies calculated under WGD and FD. Similar to Fig. 2, it shows that most efficiencies associated with the WGD model are close to zero, signaling highly efficient power production. This is in line with previous studies on US power generation, e.g., Hampf [30] and Hampf and Rødseth [10]. The FD model, on the other hand, signals substantial efficiency improvement potentials for many electricity generators. The average efficiency scores associated with the FD and WGD models are 7 and 198, respectively.

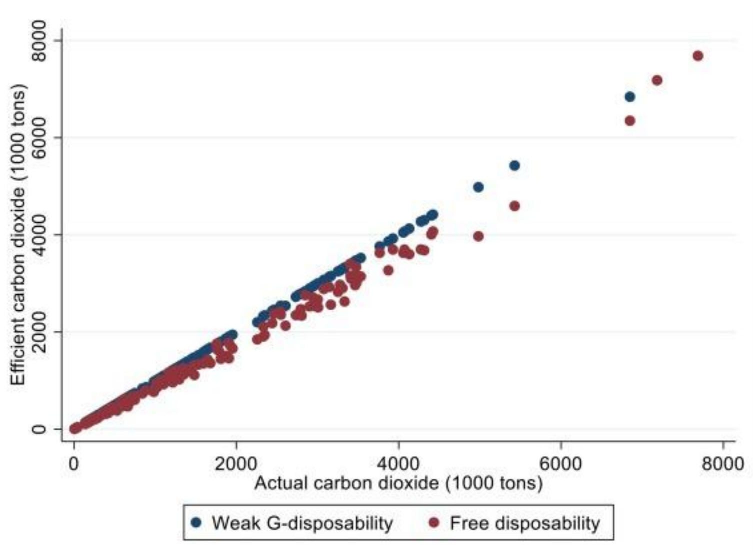


Download : Download high-res image (145KB)
Download : Download full-size image

Fig. 3. DEA efficiencies under WGD and FD. Without simultaneous estimation of effects of contextual variables.

The substantial difference between WGD and FD efficiencies – as depicted by Fig. 3 – can be associated with the FD model's lack of awareness of that carbon dioxide is closely linked to the quantity of coal used for energy production. Similar results were also obtained by Atkinson and Tsionas [5], who incorporated the materials balance in a SFA framework.

To elaborate further, Fig. 4 compares actual and efficient CO_2 emissions under WGD and FD. It clearly shows that the latter suggests minimal emissions that fall short of the materials balance condition (i.e., the 45-degree line). Hence, efficient FD CO_2 emissions can only be physically attainable if carbon contents of the coal consumed are allowed to be less than their observed levels; cf. Fig. 1. We consider this an irrelevant and unrealistic assumption.



Download : Download high-res image (182KB)
Download : Download full-size image

Fig. 4. Comparing actual and efficient carbon dioxide emissions under WGD and FD. Without simultaneous estimation of the effects of contextual variables.

4.2. Shadow prices

Using Eq. (6), we derive shadow price estimates for each of the 8 empirical models implemented, utilizing the dual price estimates of the DEA/CNLS models. Table 3 provides summary statistics of these shadow price estimates.

Table 3. Shadow price estimates ($/ton).

Disposability	Weak G--disposability				Free disposability			
Frontier	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**
Contextual vars	Without	Without	With	With	Without	Without	With	With
N	81	103	92	111	144	144	144	142
Median	12,786.01	6,908.04	6,219.40	8,361.96	71.19	77.72	72.96	73.19
Mean	40,666.00	19,041.82	58,158.17	14,415.17	69.95	76.69	71.09	73.29
Min	3,333.80	633.05	2 368.34	755.65	37.95	14.03	1.52	1.56
Max	1,079,277.00	43, 968.30	808,826.30	167,596.30	106.93	117.81	110.40	122.34

Note that shadow prices are *not* reported either when i) electricity prices are unavailable (i.e., for only up to 144 of the 160 DMUs used for estimation of the Directional Distance Function) or ii) the dual price for the electricity output is zero. As expected, the latter is primarily the case under WGD, implying that shadow prices tend to infinity due to lack of substitution possibilities among good and bad outputs when inputs are fixed.

Focusing solely on shadow price estimates that do not tend to infinity, there is a substantial difference between the magnitudes of WGD and FD shadow price estimates, regardless of frontier model (CNLS or DEA) considered and use of controls for contextual variables (With or Without). Again, this is due to the WGD model's preservation of the relationship among the pollution-generating input and CO_2, which leaves little room for reducing bads for a given input vector. Consequently, shadow prices become very high. The FD model, however, assumes less stringent ties between these variables, thereby enabling low or moderate shadow prices.

A perhaps more surprising finding is that the deterministic DEA model produces substantially lower shadow price estimates than stochastic CNLS under WGD, especially when contextual factors are not controlled. Under FD, shadow price estimates based on DEA and CNLS are of comparable sizes: DEA exhibits lower minimum values than CNLS, but medians and averages are slightly higher for DEA.

This leads to the following conclusion: Contrary to Kuosmanen and Zhou's [17] study, our results do in general not support the claim that widespread use of deterministic methods such as DEA is a key explanation of high shadow price estimates found in the literature. However, in contrast to Kuosmanen and Zhou, this study does not pay attention to local (or quantile) frontiers.

4.2.1. Robustness checks

Preceding results are all evaluated for the direction vector $(\mathbf{g}_x, g_y, g_b) = (\mathbf{0}, 1, 1)$. It is well known that empirical results – including shadow prices – are sensitive to the choice of direction vector. Appendix B consequently presents shadow price results for a set of alternative direction vectors that give disproportional weights to intended and bad outputs. These robustness checks reveal that shadow price estimates are fairly robust to the choice of direction vector under free disposability, but not under weak G-disposability: In the limiting case where the direction vector is zero for the bad output some of the shadow prices become negative and shadow prices become lower for CNLS than for DEA. Regardless, the robustness checks support the preceding finding that shadow prices are in general (substantially) higher under WGD than under FD and also higher for DEA compared to CNLS.

Further robustness checks stem from comparing our results to shadow price estimates from the literature. Kuosmanen and Zhou [17] summarize the results of 11 recent studies that report shadow prices ranging between $16 and $476 per ton of CO_2. While the FD results from this study fall within this range, shadow prices estimated under WGD largely surpass the estimates from previously published studies.

4.2.2. Replicability

One referee noted that the preceding results are contingent on a dataset that is coming of age, and that it will be preferable to reexamine key findings using more recent data. This section evaluates replicability of the main results using an alternative dataset comprising 171 electricity generating units in operation in 2015. This dataset was prepared by Hampf and Rødseth [10], and we refer to their study concerning the sources of the dataset and how it was prepared. For the replicability study, fuel and generating capacity are regarded as inputs and electricity and CO_2 emissions are regarded as outputs, while age of the facility and use of scrubbers (i.e., a dummy variable indicating use of pollution controls for sulfur dioxide emissions) are regarded as contextual variables. Appendix C presents summary statistics of the alternative dataset.

The dataset compiled by Hampf and Rødseth [10] does not contain information about the price for electricity. For the replicability test, we consider an average price of 101 $/MWh, which is obtained from U.S. Energy Information Agency's Electric Power Annual 2015. While assuming a uniform price can lead to incorrect shadow prices of carbon dioxide emission for the individual producer, it does not affect the comparison of the weak G-disposable model and the conventional production model. As this is our primary aim, we consider using an average price sufficient for the replicability check.

Table 5 summarizes the results of the replicability test. Comparing it to Table 3, it is clear that the replicability test reconfirms previous findings that shadow prices are substantially higher under weak G-disposability compared to free disposability. The reason for this is that the technical relationship among pollution-generating inputs and bads is explicitly modeled by the former approach while it is ignored by the latter.

Table 4. Effects of contextual variables on efficiencies.

	(CNLS) WGD	(CNLS) FD
Generator age	0.126*	5.740***
	(0.076)	(1.086)
SO_2/electricity	0.086	2.486
	(0.169)	(2.417)
NO_x/electricity	0.972	−7.876
	(1.003)	(14.368)
N	160	160

Standard errors in parentheses.

* $p<0.10$, ** $p<0.05$, *** $p<0.01$.

Table 5. Shadow price estimates based on an alternative dataset ($/ton).

Disposability	**Weak G--disposability**				**Free disposability**			
Frontier	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**
Contextual vars	Without	Without	With	With	Without	Without	With	With
N	131	129	128	128	171	170	171	171
Median	4,053.27	482.50	3,119.88	1,000.48	116.91	134.03	116.91	135.18
Mean	122,285.70	2,920.11	30,560.40	2,722.81	110.48	119.77	109.24	119.93
Min	93.02	53.67	61.16	18.91	44.19	27.19	19.48	26.55
Max	389,515.50	44,889.21	108,528.20	16,884.01	116.91	174.38	116.91	173.51

4.3. Effects of contextual variables

While contextual variables are primarily applied as robustness checks to consider how controlling for vintage effects and other air pollutants impact main shadow price estimates, their parameter estimates are also of interest for understanding drivers of (CO_2-focused) environmental efficiency. Table 4 presents the empirical estimates of the associated parametric functions for the CNLS WGD and FD models fitted using the dataset outlined in Section 3.

In line with Hampf and Rødseth [4], we find that newer generating units are in general more environmentally inefficient than older units. This finding is discussed in more detail by Hampf and Rødseth [4], who argue that this is likely related to the implementation of more stringent environmental regulation for NO_x and SO_2 over time. Regardless, the impacts of contextual variables are substantially less pronounced, both in terms of magnitude and statistical significance, for WGD than for FD. The explanation for this is found in Section 4.1, namely that the latter overestimates potential for efficiency improvement for the case study at hand.

5. Summary and conclusions

This paper has derived a CNLS estimator for the weak G-disposable techology. Based thereon, shadow prices for CO_2 emissions from power generation are estimated considering both stochastic and deterministic reference technologies and with and without contextual factors. All results are compared to reference technologies under free disposability of pollution-generating input and bad output.

Our empirical investigations find average efficiency scores associated with the weak G- and free disposability models equal to 7 and 198, respectively, using the Directional Distance Function. This means that the potential to reduce carbon dioxide emissions for given inputs are found negligible under the materials balance condition, while it is deemed promising under free disposability. The latter therefore leads to predicted minimal emissions that do not coincide with observed emissions. Average shadow price estimates for carbon dioxide range between 14,000 and 40,000 \$/ton CO_2 for the weak G-disposable model and between 70 and 77 \$/ton CO_2 under free disposability. The substantial shadow prices under weak G-disposability reflect the limited degree of substitutability among CO_2 and fuel consumption when taking the materials balance principle into consideration.

The empirical findings clearly illustrate that ignoring the strict dependence of CO_2 on the carbon content and therefore the quantity of coal consumed – which is the case under free disposability – leads to overestimation of environmental inefficiency and underestimation of shadow prices for CO_2. In particular, we show that the standard approach of the shadow pricing literature – to assume that bads can be reduced by lowering good outputs – becomes less relevant when the amount of bads is contingent on the fuel consumption as the latter is usually assumed fixed when calculating shadow prices. Hence, emissions are close to predetermined when there are no end-of-pipe abatement technologies present.

Kuosmanen and Zhou [17] are concerned with shadow price estimates presented in the literature that substantially surpass prices for tradeable emission permits. In line with our results, they find emission mitigation only by means of foregoing intended outputs too restrictive for abatement cost estimation. Furthermore, they argue that deterministic frontier estimation methods such as DEA overestimate marginal abatement costs because they overestimate shadow prices faced by inefficient units. To resolve the latter, they develop a quantile DEA approach that enables estimating shadow prices at “local” frontiers. In line with this approach, we compare shadow prices based on stochastic CNLS to deterministic DEA. Unlike Kuosmanen and Zhou [17], our results do in general not support lower shadow prices of the stochastic model.

Another potential explanation for deviations among shadow price estimates and quota prices that is not addressed by Kuosmanen and Zhou [17] is whether the shadow pricing literature focuses on the wrong objective function for marginal abatement cost estimation. Indeed, an intuitive approach to analyzing the economic impacts of environmental regulation is to consider how emission constraints affect DMUs’ profits; see e.g., Rødseth [28], [31]. The dual price associated with an emission constraint can readily be interpretted as marginal abatement costs, reflecting the least costly approach among available strategies for reducing emissions. A new shadow pricing approach focused on the dual of the restricted profit function can for example be built from dual formulations of the profit function developed in Kuosmanen and Kazemi Matin [32]. We leave this as a promising avenue for further research.

Finally, the empirical investigations presented in this paper use data from 2011 and 2015. While we consider that there have not been any major shifts in terms of carbon dioxide removal for the power generators under consideration, and consequently that the results are robust, we encourage further research to replicate our study using more recent data or other datasets on the joint production of intended outputs and carbon dioxide emissions.

CRediT authorship contribution statement

Kenneth Løvold Rødseth: Conceptualization, Methodology, Software, Formal analysis, Data curation, Writing – original draft.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Appendix A. GAMS code

```
SETS.

i index of dmus /i1*i160/.

k index of contextual vars /k1*k3/.

;

alias(i,j).
```

```
;

PARAMETERS.

X(i) Fuel input of firm i.

C(i) Capacity input of firm i.

Y(i) Output of firm i.

B(i) badput of firm i.

Z(i,k) contextual vars.

EF avg. emission factor /0.102899223/.

gy(i) direction vector y.

gb(i) direction vector b.

EffDEA_WGD(i) residual CNLS weak G-disposable.

EffDEA_FD(i) residual CNLS freely disposable.

EffDEA_WGD_ZVAR(i) residual CNLS weak G-disposable.

EffDEA_FD_ZVAR(i) residual CNLS freely disposable.

MRT_CNLS_WGD(i) marginal rate of transformation - weak G-disposable.

MRT_DEA_WGD(i) marginal rate of transformation - weak G-disposable.

MRT_CNLS_WGD_ZVAR(i) marginal rate of transformation - weak G-disposable.

MRT_DEA_WGD_ZVAR(i) marginal rate of transformation - weak G-disposable.

MRT_CNLS_FD(i) marginal rate of transformation - free disposable.

MRT_DEA_FD(i) marginal rate of transformation - free disposable.

MRT_CNLS_FD_ZVAR(i) marginal rate of transformation - free disposable.

MRT_DEA_FD_ZVAR(i) marginal rate of transformation - free disposable.

Yhat_CNLS_WGD_ZVAR(i) predicted frontier.

Yhat_CNLS_FD_ZVAR(i) predicted frontier.

Beta_WGD(k) Parameters for contextual variables - weak G-disposable.

Beta_FD(k) Parameters for contextual variables - free disposable.

;

* The following command assumes the data files have been saved in the roor of drive C:

$libinclude xlimportXC:\Axioms_data\WGD\Input.xls a1:fd2.

$libinclude xlimport C C:\Axioms_data\WGD\Capacity.xls a1:fd2.

$libinclude xlimport B C:\Axioms_data\WGD\Badput.xls a1:fd2.

$libinclude xlimport Y C:\Axioms_data\WGD\Output.xls a1:fd2.

$libinclude xlimport Z C:\Axioms_data\WGD\Contextual.xls a1:d161.

;

Display X,C,B,Y,Z;
```

*Set direction vectors.

gy(i)=1;

gb(i)=1;

VARIABLES.

E(i) Composite error term (v+u).

a(i) scale coefficient.

bx(i) slope coeff x.

bb(i) slope coeff b.

bz(k).

SS Sum of squares of residuals.

Chat(i) emission function.

Beta Directional Distance Function.

;

POSITIVE VARIABLES.

Epos(i) one-sided error term.

by(i) slope coeff y.

bc(i) slope coeff c.

bxpos(i) slope coeff free dispos.

bbpos(i) slope coeff free dispos.

;

EQUATIONS.

QSSE_CNLS objective function=sum of squares of residuals.

QSSE_DEA.

QREG_CNLS_WGD(i) regression equation weak G-disposable.

QREG_DEA_WGD(i) regression equation weak G-disposable.

QREG_CNLS_WGD_ZVAR(i) regression equation weak G-disposable.

QREG_DEA_WGD_ZVAR(i) regression equation weak G-disposable.

QREG_CNLS_FD(i) regression equation freely disposable.

QREG_DEA_FD(i) regression equation freely disposable.

QREG_CNLS_FD_ZVAR(i) regression equation freely disposable.

QREG_DEA_FD_ZVAR(i) regression equation freely disposable.

QCONC_WGD(i,j) curvature constraint (Afriat inequalities) weak G-disposable.

QCONC_FD(i,j) curvature constraint (Afriat inequalities) freely disposable.

QWGD(i) Weak G-dispos constraint.

QDDF_WGD(i) Directional Distance Function weak G-disposable.

```
QDDF_FD(i) Directional Distance Function free disposable.

;

QSSE_CNLS.. SS=e=sum(i,E(i)*E(i));

QSSE_DEA.. SS=e=sum(i,Epos(i)*Epos(i));

QREG_CNLS_WGD(i).. by(i)*Y(i)=e=a(i)+bx(i)*X(i)+bc(i)*C(i).

+ bb(i)*B(i) - E(i);

QREG_DEA_WGD(i).. by(i)*Y(i)=e=a(i)+bx(i)*X(i)+bc(i)*C(i).

+ bb(i)*B(i) - Epos(i);

QREG_CNLS_WGD_ZVAR(i).. by(i)*Y(i)=e=a(i)+bx(i)*X(i)+bc(i)*C(i).

+ bb(i)*B(i)+sum(k,bz(k)*Z(i,k)) - E(i);

QREG_DEA_WGD_ZVAR(i).. by(i)*Y(i)=e=a(i)+bx(i)*X(i)+bc(i)*C(i).

+ bb(i)*B(i)+sum(k,bz(k)*Z(i,k)) - Epos(i);

QREG_CNLS_FD(i).. by(i)*Y(i)=e=a(i)+bxpos(i)*X(i)+bc(i)*C(i).

+ bbpos(i)*B(i) - E(i);

QREG_DEA_FD(i).. by(i)*Y(i)=e=a(i)+bxpos(i)*X(i)+bc(i)*C(i).

+ bbpos(i)*B(i) - Epos(i);

QREG_CNLS_FD_ZVAR(i).. by(i)*Y(i)=e=a(i)+bxpos(i)*X(i)+bc(i)*C(i).

+ bbpos(i)*B(i)+sum(k,bz(k)*Z(i,k)) - E(i);

QREG_DEA_FD_ZVAR(i).. by(i)*Y(i)=e=a(i)+bxpos(i)*X(i)+bc(i)*C(i).

+ bbpos(i)*B(i)+sum(k,bz(k)*Z(i,k)) - Epos(i);

QCONC_WGD(i,j).. a(i)+bx(i)*X(i)+bc(i)*C(i)+bb(i)*B(i) - by(i)*Y(i)=l=.

a(j)+bx(j)*X(i)+bc(j)*C(i)+bb(j)*B(i) - by(j)*Y(i);

QCONC_FD(i,j).. a(i)+bxpos(i)*X(i)+bc(i)*C(i)+bbpos(i)*B(i) - by(i)*Y(i)=l=.

a(j)+bxpos(j)*X(i)+bc(j)*C(i)+bbpos(j)*B(i) - by(j)*Y(i);

QWGD(i).. bx(i)+EF*bb(i)=g=0;

QDDF_WGD(i).. by(i)*gy(i)+bb(i)*gb(i)=e=1;

QDDF_FD(i).. by(i)*gy(i)+bbpos(i)*gb(i)=e=1;

**** Stage 1: Weak G-disposable models ***.

MODEL CNLS_WGD /QSSE_CNLS,QREG_CNLS_WGD,QCONC_WGD,QDDF_WGD,QWGD/.

OPTION solvelink=0;

OPTION limrow=0;

OPTION limcol=0;

OPTION SOLPRINT=OFF;

OPTION optcr=0.0;

OPTION iterlim=10000000;
```

```
OPTION reslim=10000000;

OPTION decimals=3;

OPTION NLP=MINOS;

$libinclude gams2txt.

SOLVE CNLS_WGD using NLP Minimizing SS;

MRT_CNLS_WGD(i)$(by.l(i)>0)=bb.l(i)/by.l(i);

*.

MODEL DEA_WGD /QSSE_DEA,QREG_DEA_WGD,QCONC_WGD,QDDF_WGD,QWGD/.

OPTION solvelink=0;

OPTION limrow=0;

OPTION limcol=0;

OPTION SOLPRINT=OFF;

OPTION optcr=0.0;

OPTION iterlim=10000000;

OPTION reslim=10000000;

OPTION decimals=3;

OPTION NLP=MINOS;

$libinclude gams2txt.

SOLVE DEA_WGD using NLP Minimizing SS;

MRT_DEA_WGD(i)$(by.l(i)>0)=bb.l(i)/by.l(i);

EffDEA_WGD(i)=Epos.l(i);

*.

MODEL CNLS_WGD_ZVAR /QSSE_CNLS,QREG_CNLS_WGD_ZVAR,QCONC_WGD,QDDF_WGD,QWGD/.

OPTION solvelink=0;

OPTION limrow=0;

OPTION limcol=0;

OPTION SOLPRINT=OFF;

OPTION optcr=0.0;

OPTION iterlim=10000000;

OPTION reslim=10000000;

OPTION decimals=3;

OPTION NLP=MINOS;

$libinclude gams2txt.

SOLVE CNLS_WGD_ZVAR using NLP Minimizing SS;

MRT_CNLS_WGD_ZVAR(i)$(by.l(i)>0)=bb.l(i)/by.l(i);
```

```
Beta_WGD(k)=bz.l(k);

Yhat_CNLS_WGD_ZVAR(i)=by.l(i)*Y(i) - a.l(i) - bx.l(i)*X(i) - bc.l(i)*C(i) - bb.l(i)*B(i);

*.

MODEL DEA_WGD_ZVAR /QSSE_DEA,QREG_DEA_WGD_ZVAR,QCONC_WGD,QDDF_WGD,QWGD/.

OPTION solvelink=0;

OPTION limrow=0;

OPTION limcol=0;

OPTION SOLPRINT=OFF;

OPTION optcr=0.0;

OPTION iterlim=10000000;

OPTION reslim=10000000;

OPTION decimals=3;

OPTION NLP=MINOS;

$libinclude gams2txt.

SOLVE DEA_WGD_ZVAR using NLP Minimizing SS;

MRT_DEA_WGD_ZVAR(i)$(by.l(i)>0)=bb.l(i)/by.l(i);

EffDEA_WGD_ZVAR(i)=Epos.l(i);

**** Stage 2: Freely disposable models ***.

MODEL CNLS_FD /QSSE_CNLS,QREG_CNLS_FD,QCONC_FD,QDDF_FD/.

OPTION solvelink=0;

OPTION limrow=0;

OPTION limcol=0;

OPTION SOLPRINT=OFF;

OPTION optcr=0.0;

OPTION iterlim=10000000;

OPTION reslim=10000000;

OPTION decimals=3;

OPTION NLP=MINOS;

$libinclude gams2txt.

SOLVE CNLS_FD using NLP Minimizing SS;

MRT_CNLS_FD(i)$(by.l(i)>0)=bbpos.l(i)/by.l(i);

*.

MODEL DEA_FD /QSSE_DEA,QREG_DEA_FD,QCONC_FD,QDDF_FD/.

OPTION solvelink=0;

OPTION limrow=0;
```

```
OPTION limcol=0;

OPTION SOLPRINT=OFF;

OPTION optcr=0.0;

OPTION iterlim=10000000;

OPTION reslim=10000000;

OPTION decimals=3;

OPTION NLP=MINOS;

$libinclude gams2txt.

SOLVE DEA_FD using NLP Minimizing SS;

MRT_DEA_FD(i)$(by.l(i)>0)=bbpos.l(i)/by.l(i);

EffDEA_FD(i)=Epos.l(i);

*.

MODEL CNLS_FD_ZVAR /QSSE_CNLS,QREG_CNLS_FD_ZVAR,QCONC_FD,QDDF_FD/.

OPTION solvelink=0;

OPTION limrow=0;

OPTION limcol=0;

OPTION SOLPRINT=OFF;

OPTION optcr=0.0;

OPTION iterlim=10000000;

OPTION reslim=10000000;

OPTION decimals=3;

OPTION NLP=MINOS;

$libinclude gams2txt.

SOLVE CNLS_FD_ZVAR using NLP Minimizing SS;

MRT_CNLS_FD_ZVAR(i)$(by.l(i)>0)=bbpos.l(i)/by.l(i);

Beta_FD(k)=bz.l(k);

Yhat_CNLS_FD_ZVAR(i)=by.l(i)*Y(i) - a.l(i) - bxpos.l(i)*X(i) - bc.l(i)*C(i) - bbpos.l(i)*B(i);

*.

MODEL DEA_FD_ZVAR /QSSE_DEA,QREG_DEA_FD_ZVAR,QCONC_FD,QDDF_FD/.

OPTION solvelink=0;

OPTION limrow=0;

OPTION limcol=0;

OPTION SOLPRINT=OFF;

OPTION optcr=0.0;

OPTION iterlim=10000000;
```

```
OPTION reslim=10000000;

OPTION decimals=3;

OPTION NLP=MINOS;

$libinclude gams2txt.

SOLVE DEA_FD_ZVAR using NLP Minimizing SS;

MRT_DEA_FD_ZVAR(i)$(by.l(i)>0)=bbpos.l(i)/by.l(i);

EffDEA_FD_ZVAR(i)=Epos.l(i);

*SPECIFY THE FOLDER WHERE RESULTS ARE TO BE STORED.

$libinclude xldump EffDEA_WGD C:\Axioms_data\WGD\ShadPrice.xls EffDEA_WGD a1:ek160.

$libinclude xldump EffDEA_WGD_ZVAR C:\Axioms_data\WGD\ShadPrice.xls EffDEA_WGD_ZVAR a1:ek160.

$libinclude xldump EffDEA_FD C:\Axioms_data\WGD\ShadPrice.xls EffDEA_FD a1:ek160.

$libinclude xldump EffDEA_FD_ZVAR C:\Axioms_data\WGD\ShadPrice.xls EffDEA_FD_ZVAR a1:ek160.

$libinclude xldump MRT_CNLS_WGD C:\Axioms_data\WGD\ShadPrice.xls MRT_CNLS_WGD a1:ek160.

$libinclude xldump MRT_DEA_WGD C:\Axioms_data\WGD\ShadPrice.xls MRT_DEA_WGD a1:ek160.

$libinclude xldump MRT_CNLS_WGD_ZVAR C:\Axioms_data\WGD\ShadPrice.xls MRT_CNLS_WGD_ZVAR a1:ek160.

$libinclude xldump MRT_DEA_WGD_ZVAR C:\Axioms_data\WGD\ShadPrice.xls MRT_DEA_WGD_ZVAR a1:ek160.

$libinclude xldump MRT_CNLS_FD C:\Axioms_data\WGD\ShadPrice.xls MRT_CNLS_FD a1:ek160.

$libinclude xldump MRT_DEA_FD C:\Axioms_data\WGD\ShadPrice.xls MRT_DEA_FD a1:ek160.

$libinclude xldump MRT_CNLS_FD_ZVAR C:\Axioms_data\WGD\ShadPrice.xls MRT_CNLS_FD_ZVAR a1:ek160.

$libinclude xldump MRT_DEA_FD_ZVAR C:\Axioms_data\WGD\ShadPrice.xls MRT_DEA_FD_ZVAR a1:ek160.

$libinclude xldump Beta_WGD C:\Axioms_data\WGD\ShadPrice.xls Beta_WGD a1:ek160.

$libinclude xldump Beta_FD C:\Axioms_data\WGD\ShadPrice.xls Beta_FD a1:ek160.

$libinclude xldump Yhat_CNLS_WGD_ZVAR C:\Axioms_data\WGD\ShadPrice.xls Yhat_CNLS_WGD_ZVAR a1:ek160.

$libinclude xldump Yhat_CNLS_FD_ZVAR C:\Axioms_data\WGD\ShadPrice.xls Yhat_CNLS_FD_ZVAR a1:ek160.
```

Appendix B. Robustness checks

(See Table 6, Table 7, Table 8, Table 9).

Table 6. Shadow price estimates for $g_y=2;g_b=1$ ($/ton).

Disposability	**Weak G--disposability**				**Free disposability**			
Frontier	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**
Contextual vars	Without	Without	With	With	Without	Without	With	With
N	83	103	96	110	144	144	143	142
Median	11 916.19	6 908.04	6 140.54	8 251.28	71.01	77.01	73.44	72.74
Mean	48 396.10	19 534.44	60 584.11	12 938.17	69.25	76.62	71.14	71.62
Min	3 274.93	633.05	2 307.72	753.68	36.75	14.03	7.53	0.07
Max	1 077 780.00	477 737.30	2 672 541.00	104 875.40	106.89	117.81	110.31	122.32

Table 7. Shadow price estimates for $g_y=1;g_b=2$ ($/ton).

Disposability	Weak G--disposability				Free disposability			
Frontier	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**
Contextual vars	Without	Without	With	With	Without	Without	With	With
N	91	101	91	111	144	144	144	142
Median	14 348.77	6 908.04	5 732.40	8 288.28	71.20	77.81	73.54	74.23
Mean	8 495 319.00	17 977.83	28 366.59	14 340.90	70.21	76.89	72.08	74.14
Min	1 836.82	633.05	2 394.59	759.74	38.78	14.03	10.76	10.76
Max	235 000 000.00	431 829.40	800 879.10	159 761.10	106.95	117.81	110.47	122.37

Table 8. Shadow price estimates for $g_y=1;g_b=0$ ($/ton).

Disposability	Weak G--disposability				Free disposability			
Frontier	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**
Contextual vars	Without	Without	With	With	Without	Without	With	With
N	127	139	139	140	104	85	94	109
Median	61.85	550.26	76.68	998.68	62.96	76.86	68.10	71.85
Mean	1 523.14	1 955.82	1 799.30	2 483.45	52.00	75.75	61.66	68.75
Min	−1 627.16	−2 147.55	−1 633.03	−1 633.03	0.00	5.06	0.75	8.59
Max	67 410.03	67 418.23	69 031.06	69 031.06	106.74	117.81	109.68	121.83

Table 9. Shadow price estimates for $g_y=0;g_b=1$ ($/ton).

Disposability	Weak G--disposability				Free disposability			
Frontier	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**	**CNLS**	**DEA**
Contextual vars	Without	Without	With	With	Without	Without	With	With
N	84	102	96	111	144	144	144	142
Median	14 278.12	7 144.72	7 946.96	8 288.21	71.24	77.94	73.60	74.21
Mean	45 860.60	18 028.86	54 481.37	14 257.54	70.47	77.09	71.91	73.63
Min	3 373.42	633.05	151.93	151.93	38.74	14.03	7.37	7.40
Max	1 057 991.00	393 754.50	1 235 053.00	150 641.50	107.00	117.81	110.54	122.40

Appendix C. Summary statistics for replicability analysis

(See Table 10).

Table 10. Summary statistics.

Type	Variable	Unit	Obs	Mean	St.Dev	Min	Max
Input	Capacity	MW	171	430.6	245.9	112.5	952.0
	Fuel	1000 MMBtu	171	17,901.5	14,045.9	548.8	55,766.7
Output	Electricity	100 MWh	171	18,742.4	15,165.8	554.0	60,669.8
	CO_2	1000 Tons	171	1,668.3	1,308.8	51.1	5,144.0
Contextual	Age	Years	171	46.4	10.5	19.0	63.0

Type	Variable	Unit	Obs	Mean	St.Dev	Min	Max
	SO_2 control	Dummy	171	0.8	0.4	0.0	1.0
Price	Electricty price	$ per MWh	171	101.0	0.0	101.0	101.0

Recommended articles

Data availability

Data will be made available on request.

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