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# **A re-parameterisation of the Power Model of the relationship between the speed of traffic and the number of accidents and accident victims**

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## **ABSTRACT**

This paper presents a re-analysis of the Power Model of the relationship between the mean speed of traffic and road safety. Past evaluations of the model, most recently in 2009, have broadly speaking supported it. However, the most recent evaluation of the model indicated that the relationship between speed and road safety depends not only on the relative change in speed, as suggested by the Power Model, but also on initial speed. This implies that the exponent describing, for example, a 25 percent reduction in speed will not be the same when speed changes from 100 km/h to 75 km/h as it will when speed changes from 20 km/h to 15 km/h. This paper reports an analysis leading to a re-parameterisation of the

Power Model in terms of continuously varying exponents which depend on initial speed. The re-parameterisation was accomplished by fitting exponential functions to data points in which changes in speed and accidents were sorted in groups of 10 km/h according to initial speed, starting with data points referring to the highest initial speeds. The exponential functions fitted the data extremely well and imply that the effect on accidents of a given relative change in speed is largest when initial speed is highest.

Key words: speed of traffic, traffic accidents, mathematical models, Power Model, exponential function

## 1 BACKGROUND AND PROBLEM

The relationship between speed and road safety has long been an important topic for research. Recent studies have attempted to model the relationship mathematically, with somewhat different results. In a recent evaluation of the so called Power Model (Nilsson 2004) of the relationship between speed and road safety, Cameron and Elvik (2010) found that the exponents describing the effects on accidents of a given relative change in speed vary according to traffic environment. The exponents are lower for urban and residential roads than for motorways and rural roads. This suggests that the effects on accidents of a given relative change in speed depend on initial speed. Hauer and Bonneson (2006) and Hauer (2009) fitted exponential functions to the data provided in Elvik, Christensen and Amundsen (2004) describing the effects of changes in speed on fatal accidents and injury accidents.

The data provided in Elvik, Christensen and Amundsen (2004) have since been updated and expanded (Elvik 2009). Both these reports, with references to original studies, can be downloaded free of charge from the website of the Institute of Transport Economics ([www.toi.no](http://www.toi.no)). The most recent analyses were based on 115 studies containing a total of 526 estimates of the relationship between changes in the mean speed of traffic and changes in the number of accidents or accident victims. Analyses of the expanded data set resulted in a revision of the Power Model, as suggested by the new set of exponents listed in Table 1. The general form of the Power Model is:

$$Accidents_{after} = accidents_{before} \cdot \left( \frac{Speed_{after}}{Speed_{before}} \right)^{Exponent} \quad (1)$$

Separate exponents are fitted for accidents at different levels of severity and for injured road users at different levels of severity. The Power Model implies that the effect on accidents of a given severity of a given relative change in speed is independent of initial speed. As an example, the Power Model predicts the same percentage change in the number of fatal accidents if speed is reduced from 100 to 75 km/h as when speed is reduced from 20 to 15 km/h (in both cases speed is reduced by 25 percent). This is not very plausible, as very few accidents occurring at a speed of 20 km/h are likely to be fatal.

***Table 1 about here***

The set of exponents proposed for the Power Model in Table 1 is consistent with the idea that the effect of a given relative change in speed depends on initial speed. Nevertheless, these exponents are at best a very crude approximation to a model in which the exponents vary continuously as a function of initial speed.

Hauer and Bonneson (2006) developed exponential functions according to which the effects of a given change in speed depend on initial speed. However, their analysis was not entirely successful. In the first place, data for residential roads was discarded and not used in the analyses. In the second place, analysis was not successful for property-damage-only accidents. In the third place, analysis relied on individual data points, some of which are very uncertain. In the fourth place, the functions developed are somewhat complex and the possibility of developing

a more parsimonious version of them deserves to be explored. The functions developed by Hauer and Bonneson (2006) were formulated as follows:

$$AMF \text{ (for speed change from } v \text{ to } v^*) = e^{\alpha \left[ v - v^* + \left( \frac{\beta}{2} \right) \cdot (v^2 - v^{*2}) \right]} \quad (2)$$

AMF is the Accident Modification Factor associated with a certain change in speed. Thus, an AMF of, for example, 0.80 corresponds to an accident reduction of 20 percent. Speed is stated in miles per hour.  $\alpha$  and  $\beta$  are coefficients estimated by means of regression analysis. The exponential functions developed by Hauer and Bonneson fitted the data slightly better than the Power Model.

The objective of this paper is to continue analysis along the lines of Hauer and Bonneson. The Power Model will be compared to an exponential model in order to determine which model best fits the data. The next section explains the approach taken to analysis.

## **2 DATA AND METHODS**

### **2.1 Data aggregation**

The data base compiled by Elvik (2009) contains a total of 526 estimates of the relationship between changes in speed in changes in road safety. The largest number of estimates is found for injury accidents. Table 2 presents some summary statistics for the data.

***Table 2 about here***

Analysis relying on the data aggregation approach explained below was only feasible for fatal accidents, injury accidents and property-damage-only accidents. The other categories listed in Table 2 were not included in the analyses reported in this paper. Figure 1 shows the relationship between initial speed (km/h) and estimates of the exponent in the Power Model for injury accidents. Six outlying data points were omitted to improve the readability of the Figure. Each estimate of the exponent in the Power Model was defined as:

$$\text{Estimate of exponent} = \alpha = \frac{\ln\left(\frac{Y_1}{Y_0}\right)}{\ln\left(\frac{V_1}{V_0}\right)}$$

where  $Y_0$  is the number of accidents before a change in speed,  $Y_1$  is the number of accidents after a change in speed,  $V_0$  is speed before the change and  $V_1$  is speed after the change. The values of these variables were reported in each of the studies that were included in the data base (Elvik 2009). No clear relationship can be detected between initial speed and the value of the exponent. However, Figure 1 ignores the fact that the standard errors of the data points vary considerably; more precise data points should count for more than less precise data points. As an example, the standard error of the leftmost data point in Figure 1 is 6.15. The best estimate of the exponent is 6.82. Hence, a 95 % confidence interval ranges from – 5.23 to 18.87. Other data points are more precise and should therefore carry greater weight. Since many of the data points are imprecise, a case can be made for aggregating data points to make them more precise and suitable for analysis.

***Figure 1 about here***

In Table 3, the data for injury accidents has been placed in twelve groups according to initial speed. Each group contains estimates that refer to initial speeds in a range of 10 km/h. Eleven of these groups contain one or more estimates of the exponent in the Power Model. Thus, there were 16 estimates for initial speeds between 100 and 109.9 km/h. The mean estimate of the exponent for these 16 estimates and the standard error of the mean are also shown in Table 3. The mean estimates of the exponent were obtained by synthesising individual estimates by means of meta-analysis. Each estimate was assigned a statistical weight inversely proportional to its sampling variance and a weighted mean estimate of the exponent was developed. Technical details can be found in Elvik, Christensen and Amundsen (2004).

***Table 3 about here***

Even within each of the groups included in Table 3, estimates of the exponent vary considerably. Figure 2 show a funnel plot of the 16 estimates of the exponent referring to initial speeds between 100.0 and 109.9 km/h. The solid vertical line shows the mean estimate of the exponent (3.87). The dashed lines indicate the contours of the funnel. If the variation in the estimates of the exponent were random only, all data points ought to be located inside the contours. However, as seen in Figure 2, many data points are located outside the contours of the funnel. Even so, one can discern a tendency for estimates with small standard errors to be clustered more closely together than estimates with large standard errors. The

weighted mean estimate is close to the centre of the distribution, with seven estimates greater than the mean and nine estimates smaller than the mean.

***Figure 2 about here***

The exponents listed in Table 3 show a tendency, albeit somewhat irregular, to become smaller as initial speed becomes lower. Thus, all exponents for initial speeds above 80 km/h are greater than 3. The majority of exponents for initial speeds from 70 km/h and below are smaller than 2. Thus, aggregating the data seems to reveal a pattern that was not readily observable in the swarm of individual data points shown in Figure 1. The analyses have therefore been based on aggregated data as shown in Table 3.

## **2.2 Chaining estimates of accident modification factors**

The estimates of the exponents in each of the groups in Table 3 are based on quite different changes in speed. Thus, initial speeds in the interval from 100.0 to 109.9 km/h varied between 100.0 km/h and 106.8 km/h, with a mean of 103.0 km/h. Final speeds varied from 91.4 to 107.0 km/h. The mean final speed was 100.6 km/h. Thus, the final speed in this interval did not correspond to the initial speed in the next interval (90.0 to 99.9 km/h), which was 95.0 km/h.

It is seen, however, that the mean values for initial speeds in Table 3 (113.8; 103.0; 95.0; 84.6; 74.6; 64.5; 54.5; 45.8; 35.5) are quite close to the midpoint of each interval (115, 105, 95, 85, 75, 65, 55, 45, 35). Initial and final speeds were therefore “chained” in the following way: If initial speed was 115 km/h, final



speed was assumed to be 105 km/h. This in turn was treated as the initial speed in the next interval, and so on. Table 4 illustrates the approach. Although these values for initial and final speed are not identical to the observed values, no bias is introduced, since the mean exponent for an initial speed in the interval from, for example, 100.0 to 109.9 km/h will apply to a speed of 105 km/h and to any final speed, at least in the range found for this interval, which spanned from 91.4 to 107.0 km/h. An assumed final speed of 95 km/h is well within this range.

***Table 4 about here***

The exponents in Table 3 imply the following accident modification factor (AMF):

$$AMF = \left( \frac{\textit{Final speed}}{\textit{Initial speed}} \right)^{\textit{Exponent}} \quad (3)$$

Thus, for an initial speed of 105 km/h, the accident modification factor is:

$$AMF_{95}^{105} = \left( \frac{95}{105} \right)^{3.87} = 0.679$$

Table 4 lists the accident modification factors estimated by using the exponents from Table 3. These were linked the following way:

A start value of 100 was set for the highest initial speed. This can be interpreted both as a percentage and as a relative number of accidents. Applying the first of the accident modification factors listed in Table 4, it is estimated that when the mean speed of traffic is reduced from 115 to 105 km/h, relative number of accidents is reduced from 100 to 72.8 (0.728). The statistical weight assigned to this accident reduction is the statistical weight for a change in mean speed from

115 to 105 km/h. Proceeding to the next interval, mean speed is further reduced from 105 to 95 km/h. The accident modification factor is 0.679. Multiplying 72.8 by 0.679 gives a new relative number of accidents of 49.42. Continuing down to the lowest initial speed (35 km/h), the relative number of accidents is reduced to 4.08 at the final speed of 25 km/h. All the relative numbers of accidents are anchored to the initial value of 100. The final value of 4.08 shows that by reducing the mean speed of traffic from 115 to 25 km/h, the number of accidents is reduced by 96 percent.

The data listed in three columns for initial speed, relative number of accidents, and statistical weight in Table 4 served as input to the statistical analyses comparing the Power Model to an exponential function. The statistical analyses were performed by means of SPSS version 18 software, using the curve estimation routine.

### **3 RESULTS**

Sufficient data to permit analysis was available for fatal accidents, injury accidents and property-damage-only accidents. Table 5 presents the results of the analyses.

#### ***Table 5 about here***

It is seen that both the Power Model and the exponential function fit the data extremely well. It is recognised that this to some extent probably is an artefact resulting from the data aggregation. However, both models fit the data so well that one might wonder whether there is any difference of practical interest between the

models. The use or non-use of statistical weighting of the data points does not seem to influence the results very much.

Figure 3 shows the Power Model (weighted data) and the exponential function (non-weighted data) fitted to the relative number of fatal accidents. It is seen that the functions are distinct and clearly have different implications with respect to the effect on accidents of a given change in speed.

***Figure 3 about here***

For fatal accidents, the Power Model fits the data marginally better than the exponential function in the analysis relying on weighted data. In the analysis relying on un-weighted data, the exponential function fitted better than the Power Model. For injury accidents and property-damage-only accidents, the exponential function fitted the data slightly better than the Power Model. Figures 4 and 5 show power functions and exponential functions fitted to the data.

***Figures 4 and 5 about here***

Although the power function and the exponential function fit the data almost equally well, the functions are clearly distinct, in particular at high speeds. The exponential functions predict a much larger effect of changes in speed at high levels of initial speed than the power functions do. How much larger the effect is when relying on the exponential function, as opposed to the power function, can be determined by comparing the first derivatives of these functions. For the power function, the first derivative is:

$$\text{First derivative of power function} = \alpha \cdot \beta \cdot X^{\beta-1} \quad (4)$$

Here,  $\alpha$  is the constant term and  $\beta$  is the exponent. For the exponential function, the first derivative is:

$$\text{First derivative of exponential function} = \alpha \cdot \beta \cdot e^{\beta \cdot x} \quad (5)$$

Table 6 compares the first derivatives of the functions assessed at the listed values for initial speed. The values listed in Table 6 show the instantaneous slope of the function at a given initial speed. It can, roughly, be interpreted as the increase in the number of accidents if speed increases by 1 km/h from the listed initial speed. For fatal accidents, the exponential function predicts a much larger increase in the number of accidents at high speed than the power function. It is also seen that the dependence on initial speed of the exponential function for fatal accidents is much stronger than for the power function; in other words the curvature of the exponential function changes a lot more over the range of initial speeds than the curvature of the power function. Table 6 also shows, which is plausible according to the laws of physics, that property damage only accidents depend less on speed than both injury accidents and fatal accidents.

*Table 6 about here*

#### **4 DISCUSSION**

Hauer and Bonneson (2006) and Hauer (2009) argue that an exponential function is a better model of the relationship between speed and safety than the Power Model, principally because the effect of a given relative change in speed does not depend on initial speed according to the Power Model, which seems implausible.

However, the exponential functions fitted by Hauer and Bonneson (2006) were rather complex. An important objective of this paper was therefore to investigate whether a more parsimonious function could be fitted to the data.

To eliminate the fairly large contribution of random variation to the spread of individual data points, data were aggregated for groups of initial speed spanning 10 km/h. This greatly reduced the number of data points and removed most of the contribution of random variation. Still, the data points that were retained were sufficient to determine whether a power function or an exponential function best fitted the data. With respect to fatal accidents, the power function fitted the data slightly better than the exponential function. For injury accidents and property damage only accidents, the exponential function fitted the data better than the power function. The original data set contains data that refer to serious injury accidents and slight injury accidents. Unfortunately, these data were too sparse to apply the aggregation procedure used in this paper.

A reviewer of this paper raised concern about the very high values found for R-squared. As noted above, this could to some extent be an artefact of the high level of data aggregation. To test this, the bandwidth of the groups for injury accidents was reduced from 10 km/h to 5 km/h. The new groups for initial speed were 120 to 115 km/h, 115 to 110 km/h, etc. The number of data points thus increased from 10 to 19. Simple and weighted regressions were run on the 19 data points. R-squared was 0.957 (simple) and 0.965 (weighted) for the Power Model and 0.995 (simple) and 0.993 (weighted) for the exponential function, suggesting that the

results are robust with respect to the level of data aggregation. Figure 6 shows the functions fitted to the 19 data points.

*Figure 6 about here*

Should a power function or an exponential function be applied when estimating the effects on accidents of changes in speed? On the whole, the analyses presented in this paper lend stronger support to the use of an exponential function than to the use of a power function. It is important that the analyses presented in this paper are updated and refined as new studies of the relationship between speed and road safety are published.

## **5 CONCLUSIONS**

Data that have been used in earlier analyses of the relationship between speed and road safety have been re-analysed. The analyses support using an exponential function for modelling the relationship between speed and the number of accidents. The exponential function fits particularly well for injury accidents.

## **ACKNOWLEDGEMENT**

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Table 1:

Summary estimates of exponents by traffic environment						
Accident or injury severity	Rural roads/freeways		Urban/residential roads		All roads	
	Best estimate	95 % confidence interval	Best estimate	95 % confidence interval	Best estimate	95 % confidence interval
Fatal accidents	4.1	(2.9, 5.3)	2.6	(0.3, 4.9)	3.5	(2.4, 4.6)
Fatalities	4.6	(4.0, 5.2)	3.0	(-0.5, 6.5)	4.3	(3.7, 4.9)
Serious injury accidents	2.6	(-2.7, 7.9)	1.5	(0.9, 2.1)	2.0	(1.4, 2.6)
Seriously injured road users	3.5	(0.5, 5.5)	2.0	(0.8, 3.2)	3.0	(2.0, 4.0)
Slight injury accidents	1.1	(0.0, 2.2)	1.0	(0.6, 1.4)	1.0	(0.7, 1.3)
Slightly injured road users	1.4	(0.5, 2.3)	1.1	(0.9, 1.3)	1.3	(1.1, 1.5)
Injury accidents – all	1.6	(0.9, 2.3)	1.2	(0.7, 1.7)	1.5	(1.2, 1.8)
Injured road users – all	2.2	(1.8, 2.6)	1.4	(0.4, 2.4) #	2.0	(1.6, 2.4)
Property-damage-only accidents	1.5	(0.1, 2.9)	0.8	(0.1, 1.5)	1.0	(0.5, 1.5)

# Confidence interval specified informally

Table 2:

<b>Category</b>	<b>Number of data points</b>	<b>Highest initial speed (km/h)</b>	<b>Lowest initial speed (km/h)</b>
Fatal accidents	53	113.0	39.0
Fatalities	41	128.0	37.8
Serious injury accidents	23	107.0	37.7
Seriously injured road users	21	120.4	37.6
Slight injury accidents	23	107.0	37.7
Slightly injured road users	19	120.4	37.6
Injury accidents (severity not specified)	238	120.4	17.4
Injured road users (severity not specified)	18	128.0	39.7
Property-damage-only accidents	90	119.5	31.5

Table 3:

Range for initial speed (km/h)	Number of estimates in range	Mean initial speed (km/h)	Mean final speed (km/h)	Mean estimate of exponent (Power Model)	Standard error of exponent
120.0-129.9	1	120.4	121.2	47.20	19.75
110.0-119.9	6	113.8	101.7	3.49	1.14
100.0-109.9	16	103.0	100.6	3.87	0.98
90.0-99.9	29	95.0	91.7	3.25	0.68
80.0-89.9	34	84.6	82.4	3.42	0.72
70.0-79.9	29	74.6	72.2	1.37	0.57
60.0-69.9	37	64.5	57.6	1.12	0.55
50.0-59.9	42	54.6	48.8	1.98	0.32
40.0-49.9	24	45.8	39.3	2.18	0.35
30.0-39.9	17	35.5	30.0	1.12	0.76
20.0-29.9	0				
10.0-19.9	1	17.4	13.7	6.82	6.15
Total or mean	236	70.5	65.9	2.12	0.12

Table 4:

Initial speed (km/h)	Final speed (km/h)	Estimated AMF based on mean exponent	Relative number of accidents	Statistical weight
115	105	0.728	100.00	3.546 #
105	95	0.679	72.80	0.773
95	85	0.697	49.42	1.043
85	75	0.652	34.43	2.167
75	65	0.822	22.44	1.938
65	55	0.829	18.44	3.079
55	45	0.672	15.30	3.327
45	35	0.578	10.28	9.536
35	25	0.686	5.94	8.336
25			4.08	1.712

# Statistical weight set equal to the mean of weights applying to initial speeds between 115 and 35 km/h. These weights are shifted down one row in the table

Table 5:

Accident category	Model of analysis	Results for Power Model			Results for exponential function		
		R-squared	Coefficients	Standard error	R-squared	Coefficients	Standard error
Fatal accidents	Not weighted	0.981	Constant = 2.473 <sup>-7</sup>	0.000	0.985	Constant = 0.065	0.021
			Exponent = 4.177	0.293		Speed term = 0.069	0.004
	Weighted	0.987	Constant = 2.192 <sup>-7</sup>	0.000	0.981	Constant = 0.072	0.072
			Exponent = 4.234	0.587		Speed term = 0.069	0.012
Injury accidents	Not weighted	0.982	Constant = 0.004	0.002	0.996	Constant = 1.916	0.165
			Exponent = 2.059	0.140		Speed term = 0.034	0.001
	Weighted	0.986	Constant = 0.003	0.001	0.994	Constant = 1.983	0.083
			Exponent = 2.124	0.062		Speed term = 0.034	0.001
PDO-accidents #	Not weighted	0.989	Constant = 0.013	0.005	0.987	Constant = 3.397	0.450
			Exponent = 1.856	0.097		Speed term = 0.031	0.002
	Weighted	0.989	Constant = 0.010	0.003	0.992	Constant = 2.928	0.162
			Exponent = 1.911	0.070		Speed term = 0.032	0.001

# PDO = property damage only

Table 6:

Initial speed (km/h)	First derivative for fatal accidents		First derivative for injury accidents		First derivative for property-damage-only accidents	
	Exponential function	Power Model	Exponential function	Power Model	Exponential function	Power Model
115	12.53	6.57	3.36	1.32	3.71	1.44
105	6.28	4.85	2.39	1.19	2.70	1.33
95	3.15	3.48	1.70	1.06	1.96	1.21
85	1.58	2.40	1.21	0.94	1.42	1.09
75	0.79	1.59	0.86	0.82	1.03	0.98
65	0.40	0.99	0.61	0.70	0.75	0.86
55	0.20	0.57	0.44	0.58	0.54	0.74
45	0.10	0.29	0.31	0.46	0.40	0.61
35	0.05	0.13	0.22	0.35	0.29	0.49
25	0.03	0.04	0.16	0.24	0.21	0.36
Ratio 115/65	31.50	6.66	5.47	1.90	4.95	1.68
Ratio 65/25	15.80	23.95	3.90	2.93	3.60	2.39

Figure 1:

### Relationship between initial speed (km/h) and value of exponent in the Power Model for injury accidents

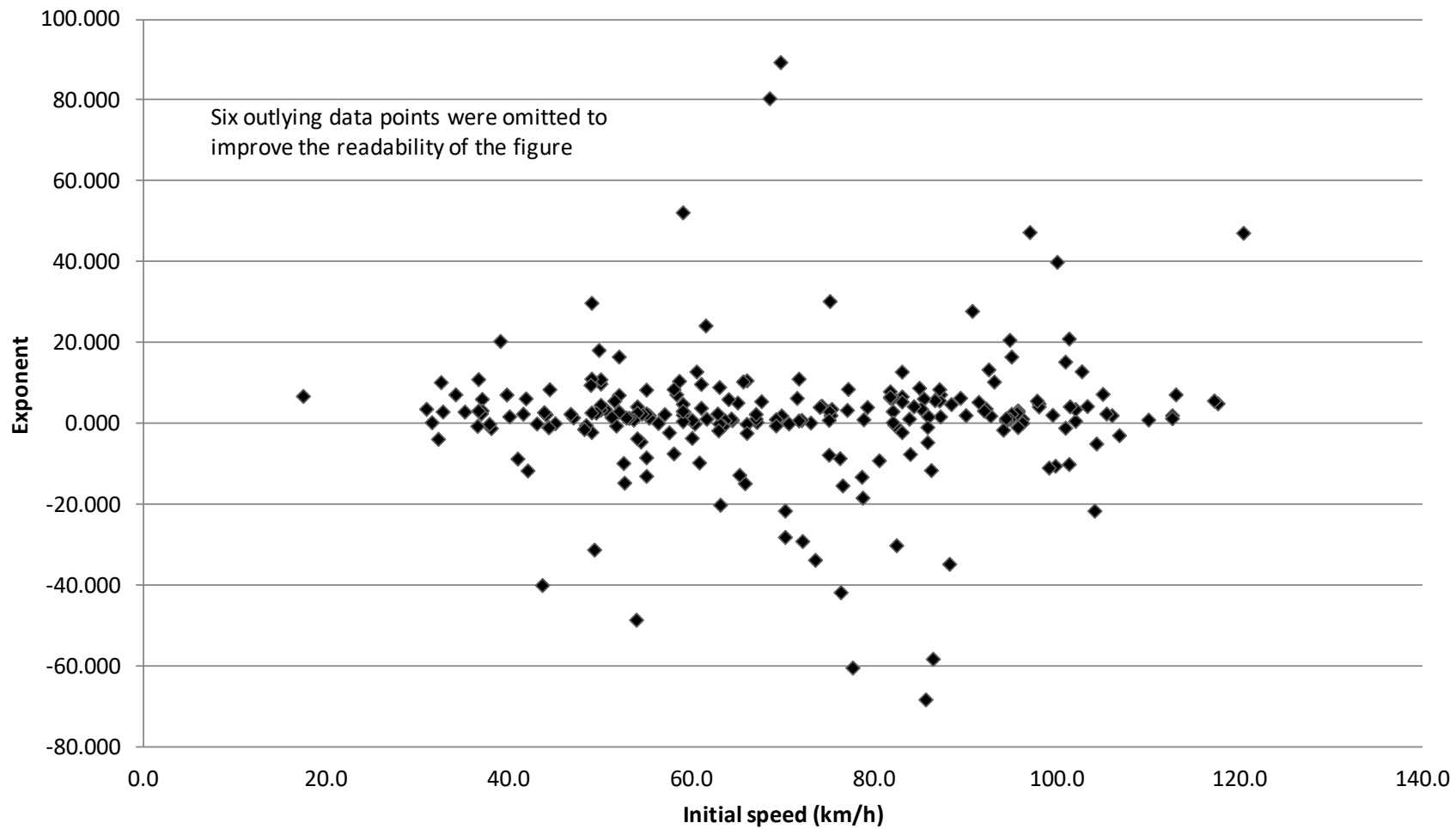




Figure 2:

### Funnel plot of estimates of exponent for initial speed between 100.0 and 109.9 km/h - injury accidents

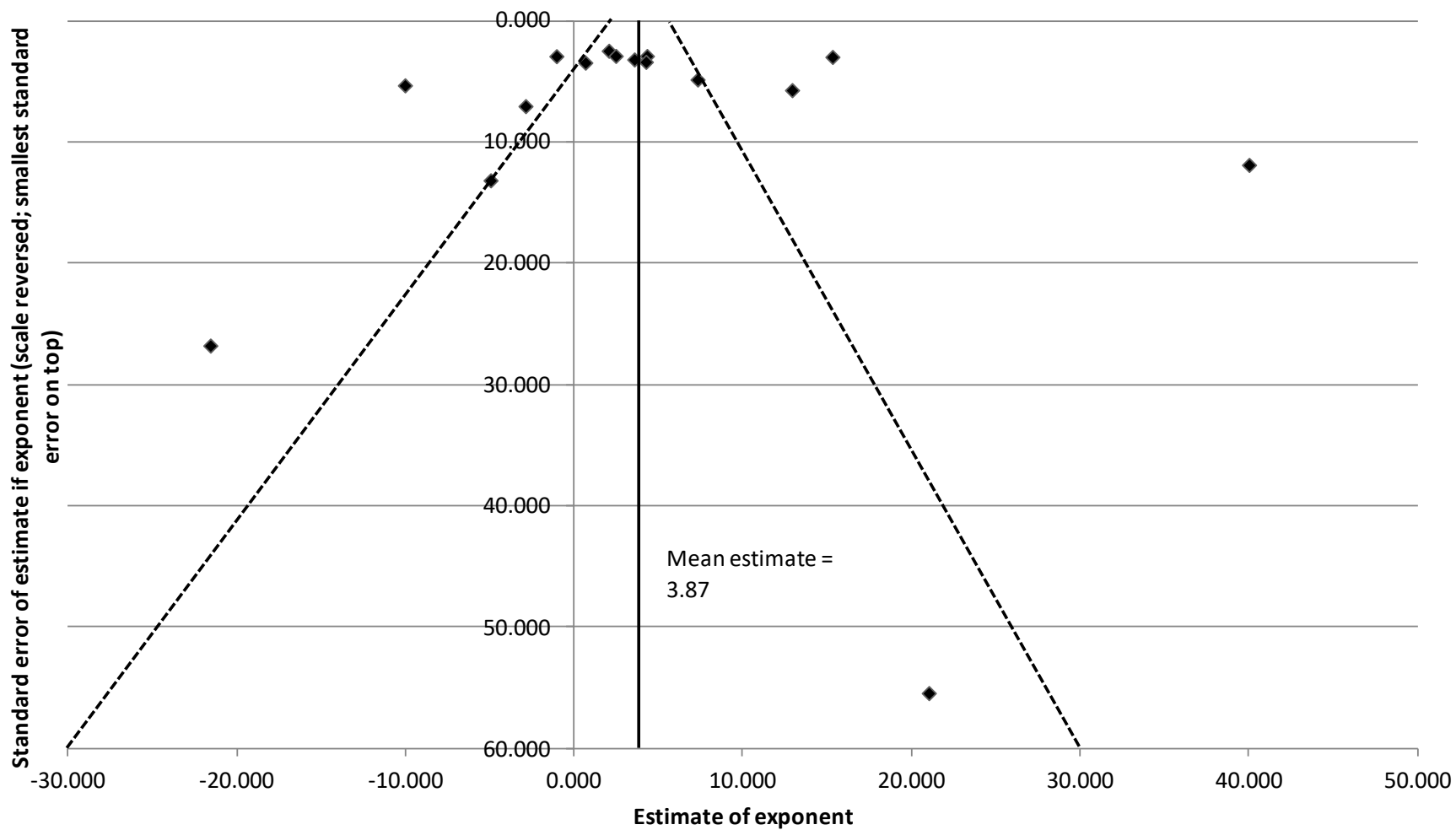


Figure 3:

### Power Model and exponential function fitted to data for fatal accidents

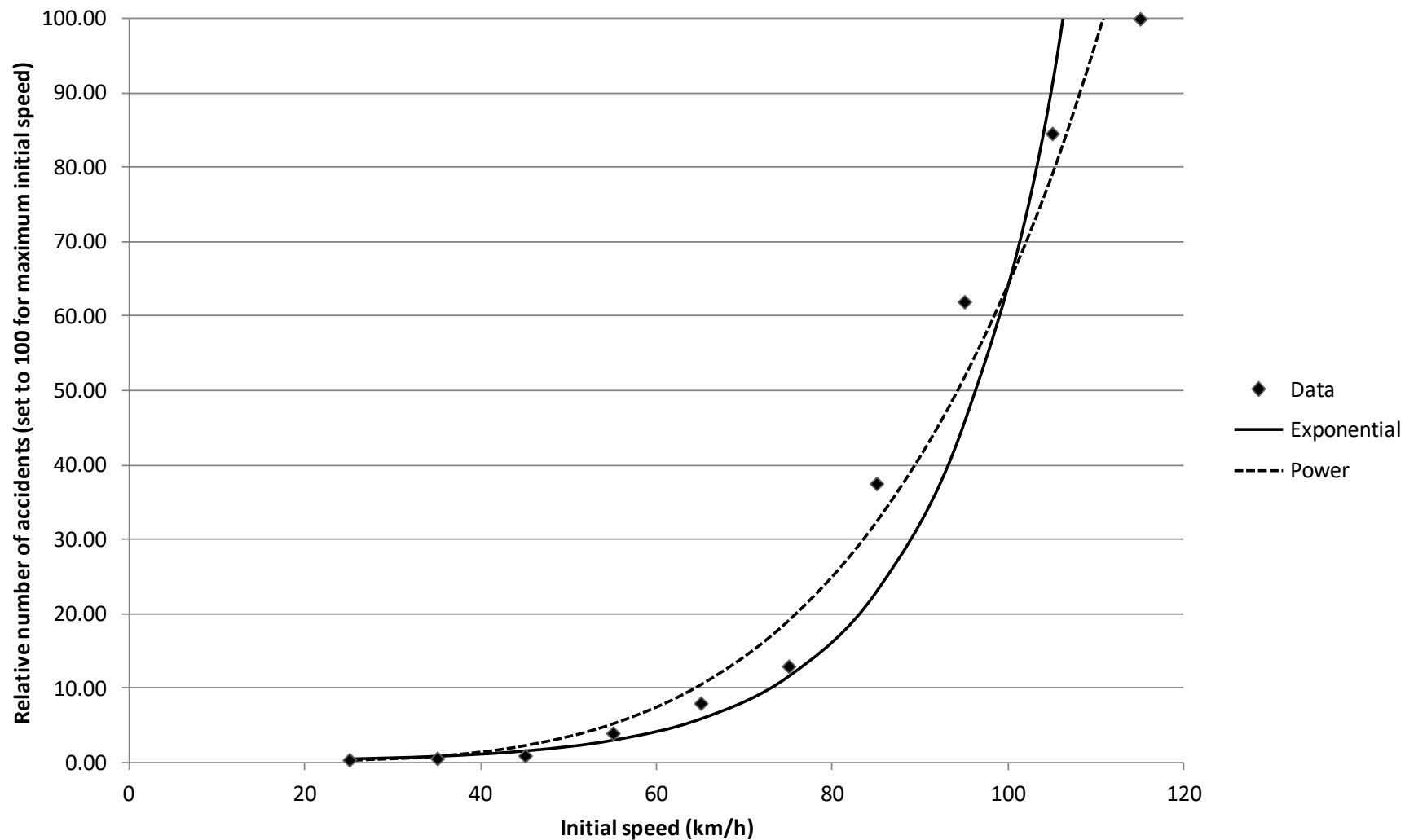


Figure 4:

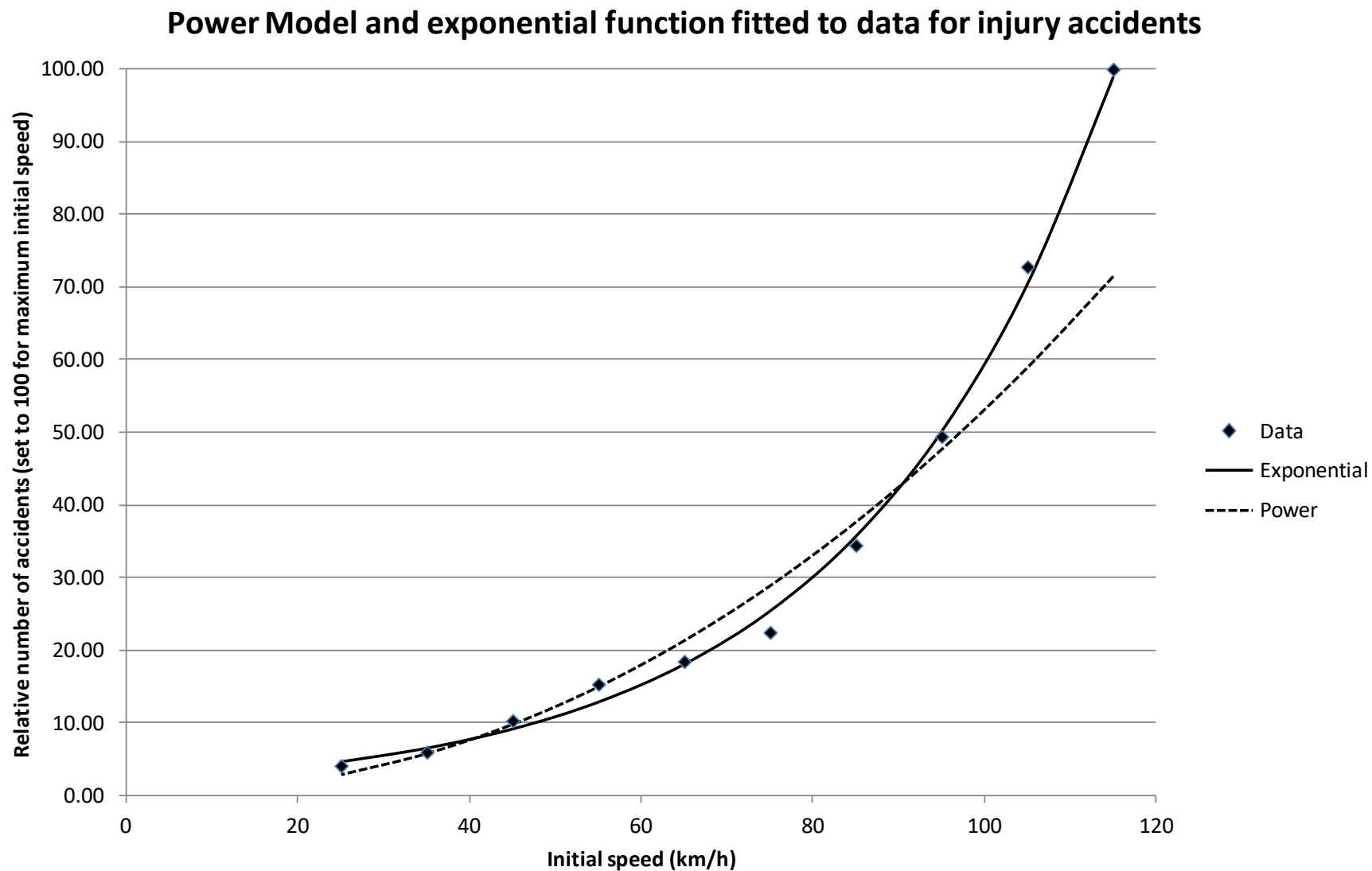


Figure 5:

### Power Model and exponential function fitted to data for property damage only accidents

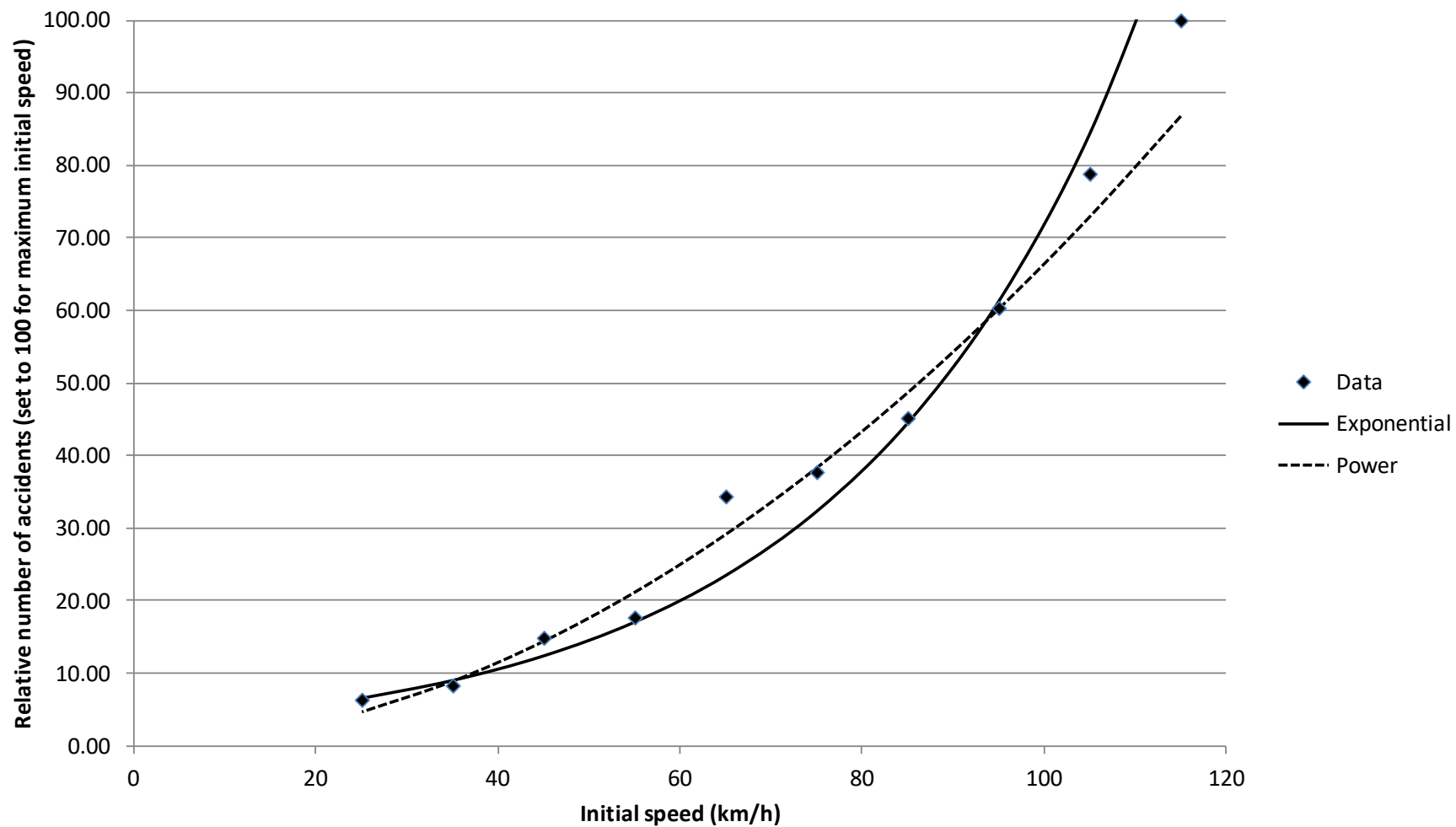


Figure 6:

### The Power Model and an exponential function fitted to 19 data points for injury accidents as a sensitivity analysis of data aggregation

